PHYSICS 6L

CLASSICAL MECHANICS

SUMMER QUARTER, 2024

LABORATORY MANUAL

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CLASSICAL MECHANICS

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INTRODUCTION

Welcome to Physics 6L, the Classical Mechanics laboratory. The goal of this laboratory is to give you hands-on experience with real mechanical systems, to help you gain an intuitive grasp of the concepts introduced in the accompanying lecture course.

You may find the format and style of the physics labs quite different from your typical chemistry or biology laboratory. In chemistry and biology, you are expected to learn and retain specific techniques, such as quantitative and qualitative analysis, spectroscopy, and so forth. By contrast, the physics lab is *not* a techniques laboratory; it is a *concepts* laboratory. Although you will learn many important techniques (such as the proper use of an oscilloscope), the overwhelming stress is on understanding the deep physical principles that govern the physical world. A firm understanding of these principles should make it much easier for you to learn the technique-intensive subjects that you are also studying.

You will also find that the manual dwells at some length on the theoretical ideas behind the experiments. This is because it is often impossible to schedule the laboratory to closely follow the lecture material; sometimes the labs actually precede the lectures. Often you might miss the lecture, in any case. This is also why the laboratory instructor is asked to spend time at the beginning of each lab explaining both the theory and the techniques being used.

Course Registration

Although this course must be taken concurrently with Physics 6A, it is a separate course, for which you must register. Decide carefully which section you want to attend, because, due to space limitations, it may not be possible for you to change sections later in the course. Make sure that the section that you actually attend is the one in which you have enrolled, or your grade may be in jeopardy.

Laboratory Sections

Each laboratory section will meet once per week. There will be about 20 students in each section. Each section will be led by a Teaching Assistant, who is likely a graduate student in Physics.

In order to get credit for your work, you must attend the laboratory section in which you are officially enrolled. *You will only get credit for attending another section ("crashing") if you have written permission, in advance, from both teaching assistants. In this case you must turn in your report to the TA at the end of the lab session, and ask him or her to deliver it to your regular TA.*

Notebooks

Please purchase the notebook that the bookstore has put aside for the physics introductory laboratories. It is a horizontally bound (left margin) notebook with vertical and horizontal (quadrille) rulings, about 60 pages and approximately 8 inches by 10 inches, and without perforations. **Please enter the course title, the year and quarter, and the TA's name on the inside of the front cover**.

The following records are **not** acceptable:

1. Loose-leaf binders
2. Spiral-bound notebooks
3. Vertically bound (top margin) notebooks
4. Notebooks with page perforations
5. Notebooks that have linear ruling rather than graph-style quadrille ruling.

Please note that, after the first lab section, your work will be marked down if you turn it in on loose sheets of paper, or in a notebook that is not acceptable.

Leave your notebooks with your TA at the end of each lab section, so that he or she can evaluate your work. If you don't understand or agree with your TA's comments, discuss them with him or her. For detailed guidance, please refer to **Guidelines for Laboratory Reports** in the Appendices.

Lab Partners

Work with a partner on each of the experiments. Since report preparation is an important part of the laboratory work, each of you should prepare your own notebook. Although you should feel free to refer your reader to a partner's notebook for a table of raw data, *etc.*, your calculations, descriptions, comments, and conclusions should be independently recorded.

Include your partner's name when you describe an experiment, and always enter the date. It will make it easier for your instructor. Try to choose a different partner each week. It's a chance to make new friends and to stimulate new thinking.

Pre-laboratory Questions

You will find pre-laboratory questions at the end of each chapter of this manual. These questions are intended to prepare you for the concepts and calculations that you will need for the laboratory, and are guaranteed to save you a significant amount of lab time. For this reason, it is a good idea to keep a copy of your prelab solutions with you as you do the lab. *Please submit solutions to the questions to the TA on a separate sheet of paper when you report to the laboratory class.* Late submissions will not be accepted for credit.

Warning. If we have evidence that you have copied solutions from the internet or other sources, you will receive a grade of 0 for the entire experiment's prelab.

Grades

Your work will be appraised with a letter grade. Your grade will be based upon all of the scheduled laboratories. If you miss one lab for a documented medical or family emergency, *and you notify your TA in advance and in writing*, your grade will be averaged over the remaining laboratories. If you otherwise miss a single laboratory, your final grade will be averaged over the remaining labs, but then reduced by a full letter grade. If you miss two labs for a documented medical or family emergency, *and you notify your TA in advance* *and in writing,* you may, before the end of the last week of instruction, petition in writing for an incomplete (I). If you miss two labs for any other reason, you will not pass the course.

Laboratory Etiquette

As a courtesy to your fellow students, when you are finished with the laboratory, please make sure that all of the equipment is intact and organized in an orderly way. If any equipment is missing or not working properly, please notify the instructor.

Also, with the exception of water, please do not bring food or drink into the laboratory. You may leave such items on the shelves in the hallway to consume elsewhere. Finally, by order of the Fire Marshall, under no circumstances may bicycles be brought into the laboratory. Bike racks are available at either end of the building.

Final Comments

We hope that you enjoy this lab course, and that you will find that your discoveries will be repeated again and again in the real world. We also encourage your feedback on the curriculum, the quality and readiness of the equipment, and on the laboratory manual.

Good luck!

George Brown
Professor of Physics

MEASUREMENT AND UNCERTAINTY

BACKGROUND

The array of empirical observations of the natural world that are repeatable and that can be verified by other observers is the foundation common to all scientific disciplines. Scientists strive to organize these observations into patterns, which we call "theories." A valid theory must offer predictions that can be tested. The scientific method strives to discover the simplest and most elegant patterns that are consonant with all of the observations.

For many scientific disciplines, observations can be represented in a quantitative manner. However, since no scientific instrument that measures a continuous quantity is exact, repeated observations of a given quantity will yield a slightly different result for each measurement, because we cannot perfectly control the measurement conditions. As a consequence, we have to define exactly what we mean by a repeatable or reproducible experiment. In this section we will show you how to organize a set of observations and assign an experimental uncertainty (sometimes misleadingly called the experimental error) to the measurements. We will also show you how to account for possible systematic errors (as opposed to statistical fluctuations).

EXPERIMENT

At your bench you will find a very simple apparatus. It consists of a cylindrical pendulum bob suspended from a pivot that allows one to measure the angle of the pendulum string at any moment. In this experiment we will test the following two hypotheses:

Hypothesis 1: If the bob of a pendulum of fixed length L is released at rest from an angle θ from the vertical, it will reproducibly accelerate to a certain speed v at the bottom of its travel.

Hypothesis 2: The speed of the pendulum at the bottom of its travel will be given by the formula

$$v = 2\sqrt{gL}\,\sin(\theta/2) \tag{1}$$

Where L is the pendulum length measured from the pivot to the center of mass of the bob, and where g is the earth's acceleration of gravity at the laboratory. In Santa Cruz, $g = 9.798 \pm 0.001$ m/s^2. The physical basis for this formula will be derived in the Conservation of Energy laboratory. Our experiment will be to release the bob from a fixed angle (say 10 degrees) and make repeated measurements of the speed of a brass bob at the bottom of its travel.

Since this is your first college physics laboratory experiment, we will make it easy for you by showing you a sample data set and a sample analysis. However, we will of course expect you to carefully experiment yourselves, and test the hypotheses with your own data.

SAMPLE DATA

This data set consists of ten trials with the following parameters:

Pendulum length L = 0.50 m.
Bob material: Brass
Bob effective diameter: 0.0149 m (see footnote)
Bob starting angle: 10 degrees

TRIAL	ΔT (s)	v_i (m/s)	(v_i - <v>) (m/s)	(v_i - <v>)^2 (m/s)^2
1	0.0386	0.3860	-0.0031	0.00001
2	0.0362	0.4116	+0.0225	0.00051
3	0.0386	0.3860	-0.0031	0.00001
4	0.0362	0.4116	+0.0225	0.00051
5	0.0402	0.3706	-0.0185	0.00034
6	0.0396	0.3763	-0.0128	0.00017
7	0.0376	0.3963	+0.0072	0.00005
8	0.0400	0.3725	-0.0166	0.00028
9	0.0363	0.4105	+0.0214	0.00046
10	0.0403	0.3697	-0.0194	0.00038
Σ		3.8911	+0.0000	0.00270

Using the photogate timer, we made ten measurements of the time ΔT that it takes for the bob to pass through the photogate timer. These times are shown in column 2. For understandable reasons, we see that we get a slightly different value for ΔT each time.

In column 3, we calculate the speed of the bob in m/s at the bottom of its swing by dividing the effective diameter of the bob, 0.0149 m, by the passage time ΔT. We then calculate the average speed <v> by summing the ten values, and dividing by 10. In this example, <v> = 0.3891 m/s.

The fourth column displays the deviation of a given speed measurement from its average value; *i.e.* (v_i-<v>). As you would expect, the numbers fluctuate both positively and negatively. Furthermore, if the arithmetic is done correctly, the deviations should sum to exactly zero (as you have proved in the first prelab question). The last row in our table shows that in this case they do in fact sum to zero.

Our next task is to assign a numerical value to the experimental fluctuations. The universal measure of the experimental fluctuations is the so-called *sample standard deviation*, σ. The sample standard deviation, motivated by the theory of probability, is a convenient measure of the fluctuations of the data about the average. The sample standard deviation is defined by the following formula:

$$\sigma \equiv \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(v_i - \langle v \rangle)^2} \qquad (2)$$

This formula requires a bit of explanation. *N* refers to the number of measurements; in this example *N* = 10. We calculate the sum of the squares of the deviations, *and then divide by one less than the number of measurements*. Although this may seem strange, it comes about because <*v*> is not the true, universally correct speed, but is rather the value computed from our data. We say that the number of *degrees of freedom*, originally 10, has been reduced to 9 because we have used the data to calculate <*v*>. If we had put in a value for <*v*> that was independent of the experiment, say if it were determined from some formula or some table of physical constants, then the (*N* - 1) would be replaced by *N*.

Column 5 displays the square of the deviation for each trial, and the final entry is the sum of the squares of the deviations. Dividing this number, 0.00270, by 9, and taking the square root, we get σ = 0.0173 m/s.

The sample standard deviation, σ, is a measure of the fluctuations of our sample of *N* measurements. If we made a much larger number of measurements, say *N* = 1000 instead of *N* = 10, we would get approximately the same value for σ, because the *spread* has not changed. However, if we made such a large number of measurements, it stands to reason that the uncertainty of our final result, <*v*>, would get progressively smaller. The *standard deviation of the mean*, $\sigma_{<v>}$, is given by

$$\sigma_{<v>} = \frac{\sigma}{\sqrt{N}} \tag{3}$$

So in our experiment, $\sigma_{<v>}$ = 0.0173/3.162 = 0.0054 m/s. Our final result is expressed as follows:

$$< v >= 0.3891 \pm 0.0054 \text{ m/s} \tag{4}$$

Notice that we have expressed the standard deviation to two significant figures; since it is an uncertainty it cannot be more precise than that! Also, we have expressed the value of <*v*> to the corresponding level of accuracy.

We are now in a position to test hypothesis 2, given by eq. (1), using g = 9.798 m/s^2:

$$v_{pred.} = 2\sqrt{gL}\,\sin(\theta/2) = 0.3858 \text{ m/s} \tag{5}$$

We see that our predicted value of the speed falls fairly close to the experimental value, and certainly within one standard deviation. We conclude that our data is at least consistent with the formula given by hypothesis 2.

PROPAGATION OF ERRORS

In comparing experiment with theory in the preceding example, we implicitly assumed that we knew the values of g, L, and θ exactly. However, suppose our values were off by some systematic amount. For example, suppose we trust the reading of our meter stick to an accuracy of ±1 mm. If we calculate the predicted value of v from equation (1) using $L = 0.500 \pm 0.001$ m, we get $v_{pred} = 0.385816 \pm 0.00038$ m/s. (You can do this calculation with a hand calculator, or you can use fancy calculus methods). The point is that the discrepancy between our measured value of v and our theoretical value of v can arise from two sources: the *random* variations of the actual measurements, which we have already discussed, and the *systematic* errors in the parameters like L.

ACTUAL EXPERIMENTS

Experiment 1 – *Velocity of pendulum bob*

1) Measure the length L of your pendulum and predict the velocities for $\theta = 10$ and $\theta = 20$ degrees from Equation (1).

2) Measure the velocity of the brass pendulum bob when it is released from an angle of 10 degrees (don't forget to correct for the width of the photogate timer beam[1]). Take 10 measurements of the velocity.

3) Calculate:
    a. The mean velocity, $<v>$
    b. The standard deviation of the measurements, σ
    c. The standard deviation of the mean, $\sigma_{<v>}$

4) Repeat steps (1) and (2) for a starting angle of 20 degrees.

5) Compare your experimental results with your theoretical predictions for each angle. How far is the mean velocity from the predicted velocity, both in absolute terms and as a multiple of the standard deviation of the mean? Is the agreement satisfactory? If not, can you identify possible sources of the discrepancy?

Experiment 2 – *The effect of systematic errors on the velocity*

1) Estimate the following:
    a. The precision of your measurement of the length L of the pendulum (measured with the meter stick from the pivot point to the center of the cylindrical mass)
    b. The accuracy of your measurement of the angle θ.
2) For the 10 degree measurements, calculate the effect that these (systematic) errors would have on your predicted value for v. (Hint: use the ideas described above under the heading Propagation of Errors).

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[1] The effective width of the bob will be the actual width, corrected by the number written on the individual photogate timer. This correction arises from the fact that the bob does not trigger the photogate beam at the exact center of the beam. If the correction is a negative number, it means that the effective bob diameter is smaller than the actual bob diameter.

Experiment 3 (Optional) – *Effect of photogate correction*

The correction to the bob diameter arising from the finite width of the photogate beam may vary from timer to timer.

1) If you were to use a different correction than the value noted on the timer, would it bring both your 10 degree and 20 degree measurements of v into closer agreement with the theory? What would be the value of the correction in mm?

PRELAB QUESTIONS

1. The average value of N measurements of a quantity v_i is defined as

$$< v > \equiv \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{N}(v_1 + v_2 + \cdots v_N) \tag{6}$$

The deviation of any given measurement v_i from the average is of course (v_i - <v>). Show mathematically that the sum of all the deviations is zero; *i.e.* show that

$$\sum_{i=1}^{N}(v_i - < v >) = 0 \tag{7}$$

(Hint: break up the left hand side into two separate sums).

1. (Alt.) If you had trouble with problem 1, then do the following instead: Pick five numbers at random; take their average; tabulate the deviation of each number from the average; and add the deviations. Your result should be exactly zero.

2. Evaluate the formula in equation (1) for the following values: L = 0.500 m, g = 9.798 m/s^2, and θ = 10 degrees. Now, evaluate the same formula three more times, each time varying one of the three parameters L, g, and θ (and only one) by +1% of the given value. In each case, what is the percentage change of the final result for v?

This important exercise tells you how an error in the measurement of an experimental quantity *propagates* through to the final result. It is a general method for determining the sensitivity of an equation to its constituent parameters.

KINEMATICS

BACKGROUND

The study of an object's motion, whether the object is a dust particle or an entire galaxy, involves fundamental concepts of time and space. *Kinematics* is the language and set of mathematical techniques used to *describe* an object's motion, irrespective of the *causes* of the motion. In this way, kinematics is the logical foundation of *dynamics*, the study of the forces that cause the motion. The laws that govern the universe are elegant and powerful, but their simplicity is not always obvious until one takes the time to precisely observe and describe nature using the techniques of kinematics.

The goal of this week's lab is to understand the core concepts of kinematics—time, position, velocity, and acceleration—and the relations among them. These ideas were first clearly understood by Galileo in the early part of the seventeenth century, and we will borrow many of his experimental techniques. Galileo studied the motion of a bronze ball rolling along a wooden trough about 18 feet long. We will do something similar, replacing the ball with a glider that can coast smoothly with very little friction along an air track that is nearly two meters long.

Position and Time

All of kinematics is built upon the concepts of position and time. A position in space is normally described with respect to a three-dimensional coordinate system. In today's lab, though, we will study objects confined to move in one dimension only. In one-dimensional motion, an object's position can be described by a single number x, which tells how far the object is along its track from an arbitrarily chosen origin. One direction is defined to be positive, and the opposite direction is defined to be negative.

The displacement Δx represents the change in an object's position during some time interval. For example, if an object moved from $x_1 = 25$ cm to $x_2 = -15$ cm during some interval, then its displacement would be $\Delta x = x_2 - x_1 = -40$ cm. In other words, it moved 40 cm to the left during this time interval.

Velocity: Average and Instantaneous

Velocity, like displacement, is a vector quantity, but for one-dimensional motion it can be described using a single (signed) number. The units of velocity are those of distance divided by time, such as miles per hour or meters per second.

Suppose that we measure an object's position x_1 at some time t_1, and then measure its position x_2 at a later time t_2. The object's *average* velocity during the time interval from t_1 to t_2 is defined as the ratio of the displacement to the elapsed time:

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad (8)$$

The average velocity can be positive or negative, depending upon the direction that the object moved. An object's *average* velocity during an interval is *zero* if it ends the interval in the same place where it started, even if it clearly moved during the interval!

While the average velocity $\bar{v}$ describes an object's net motion, the instantaneous velocity $v(t)$ tells how fast the object is moving at some particular moment of time t. The instantaneous velocity is equivalent to taking the average velocity during a vanishingly short time interval around the instant t in question:

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt} \tag{9}$$

Again, the instantaneous velocity v can be positive or negative, depending on the direction the object is moving. If v is zero at some point in time, then the object is not moving at that instant.

Acceleration: Average and Instantaneous

It often proves important to track the way an object's velocity changes over time. This motivates the definition of *acceleration* as the rate of change of velocity. Acceleration is a vector, as is position and velocity. For objects moving in one dimension, acceleration can, like velocity, be described by a single number, which may be positive, negative, or zero. The units of acceleration are those of velocity divided by time, or position divided by time squared; *e.g.* meters per second squared.

During some time interval Δt, an object's *average* acceleration $\bar{a}$ is given by

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \tag{10}$$

Note that if the object begins and ends the interval moving at the same velocity, its average acceleration is zero, even if its velocity changed several times during the interval and even if it ends up in a different position.

The instantaneous acceleration $a(t)$ is defined in much the same way as was the instantaneous velocity. It tells how the velocity is changing at some particular instant:

$$a(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2} \tag{11}$$

If you have understood these concepts, you should be able to convince yourself that whenever an object's acceleration has the *same* sign as its velocity, it is speeding up. If $a(t)$ and $v(t)$ have *opposite* signs, the object is slowing down.

It is our great fortune that the fundamental laws of physics have no use for derivatives of higher order than the acceleration.

Kinematics at Constant Acceleration

An important special case of motion in one dimension is that of an object whose acceleration remains *constant* during some period of time. To be specific, we will designate this constant acceleration as a_0. This case is interesting because this is the situation for bodies in a constant gravitational field, such as for bodies near the surface of the earth.

We remind you of the definition of acceleration:

$$a(t) \equiv \frac{dv(t)}{dt} \qquad (12)$$

Recall from your calculus class that integration is just the opposite of differentiation. For this reason we can invert Equation (12), and get

$$v(t) = v(0) + \int_0^t \frac{dv}{dt} dt' = v(0) + \int_0^t a(t')dt' = v(0) + a_0 \int_0^t dt' = v_0 + a_0 t \qquad (13)$$

where we have defined v_0 to be $v(0)$, the velocity at $t = 0$. The key step was to pull $a(t')$ out from under the integral sign, because it is a constant equal to a_0 in this case. Note also that it is easy to check the equation by simply taking the time derivative.

We can now repeat this step, to determine the position of the object at any moment in time:

$$x(t) = x(0) + \int_0^t \frac{dx}{dt} dt' = x(0) + \int_0^t v(t')dt' = x(0) + \int_0^t (v_0 + a_0 t')dt' = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \quad (14)$$

where again we have defined x_0 to be $x(0)$, the position at $t = 0$. Note again that it is easy to check this equation by simply taking the time derivative.

So far, we have an expression for the velocity and the position as a function of time. For some purposes, it is useful to re-formulate the results, so that we have the position as a function of the *velocity* rather than position as a function of *time*. To do this, we see from Equation (13) that time can be expressed as a function of velocity by $t = (1/a_0)(v\text{-}v_0)$. If we substitute this expression for time into the equation for the position, we get

$$x(t) = x_0 + v_0 \frac{1}{a_0}(v - v_0) + \frac{1}{2} a_0 \left[\frac{1}{a_0}(v - v_0) \right]^2 = x_0 + \frac{1}{2a_0}(v^2 - v_0^2)$$
$$\text{or, } 2a_0 (x - x_0) = v^2 - v_0^2 \qquad (15)$$

We can summarize these results *for constant acceleration* a_0 in one neat box:

$$
\boxed{\begin{aligned} v(t) &= v_0 + a_0 t \\ x(t) &= x_0 + v_0 t + \frac{1}{2} a_0 t^2 \\ 2a_0(x - x_0) &= v^2 - v_0^2 \end{aligned}} \qquad (16)
$$

Apparatus

The main piece of apparatus in this week's lab is the two-meter long air track shown in Figure 1. When you attach an air hose to one end of the track, the air is forced out of small holes along the length of the track. These air streams are strong enough to support a glider which can then move along the track with almost no friction. A length scale is marked along one side of the track, which can be used to measure the glider's position. *Please do not make any marks on the air track!*

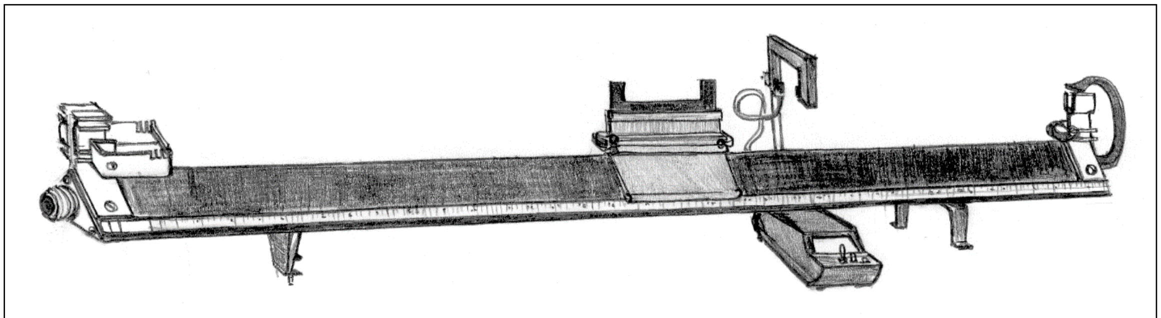

Figure 1. The air track with glider and photogate.

The air track is supported at one end on a leveling screw. When you want the track to be level, simply turn on the air, placed the glider on the track, and adjust the screw until the glider does not tend to slide in either direction when released.

You can track the glider's motion with a photogate timer, illustrated in Figure 2. The timer is triggered each time an opaque object, such as the flag on the top of the glider, blocks the photogate's infrared beam. To help you set up your experiments, a red light near the beam detector will light whenever the beam is blocked.

The timer's controls determine how it reacts to an object blocking its beam. The controls have the following actions:

RESET: This pushbutton resets all registers.

START/STOP: This pushbutton is one means of starting or stopping the timer. When the button is pressed for some time interval Δt, it is equivalent to blocking the light beam for the interval Δt (see below).

MEMORY: If one wishes to record two successive time intervals, set the **MEMORY** switch to **ON**. Ordinarily, the readout will display the first time interval. If the **MEMORY SWITCH** is on, then a hidden register keeps track of the sum of the times of two successive intervals. This hidden register may be viewed by holding the **MEMORY** switch in the **READ** position.

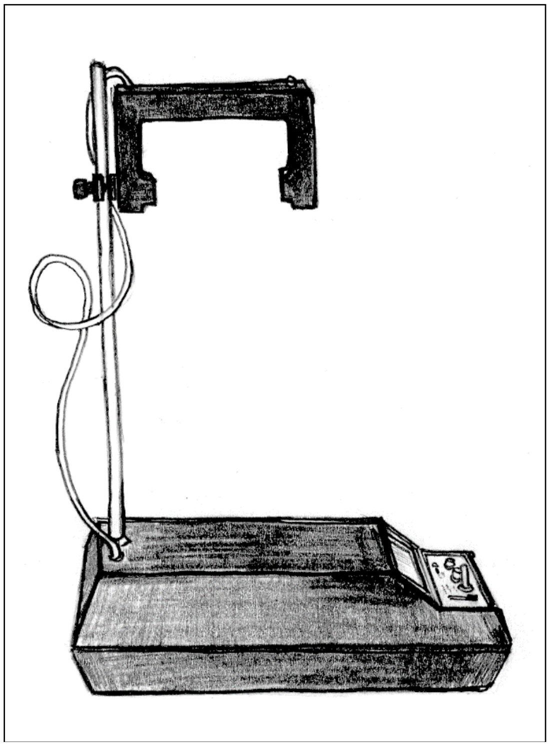

Figure 2. The photogate timer.

Important note: It is a good idea to always keep the **MEMORY** switch **ON**. If you do so, the first register of the photogate timer will not be affected by subsequent interruptions of the light beam. However, the second, "hidden register," will unfortunately be dependent upon the sequence of subsequent interruptions. Therefore, if you plan to use the contents of the hidden register, be sure to not allow the flag to interrupt the beam more times than necessary to make your reading.

The leftmost control of the timer is a four-position mode switch. The four positions are **OFF**, **GATE**, **PULSE**, and **PEND**. Figure 3 displays the tie interval displayed on the register for each of the modes described below.

OFF: Power to the timer is switched off.

GATE: If the light is first blocked, then unblocked, the register displays the elapsed time between the blocking and unblocking of the light.

PULSE: The timer is started the moment that the light is first blocked; the timer is then stopped at the moment when the light is blocked a second time.

PEND: This is the 'pendulum' mode. It is similar to the **PULSE** mode. The timer is started the moment that the light is first blocked; but the timer ignores the second blocking of the light, and then stops at the third blocking of the light.

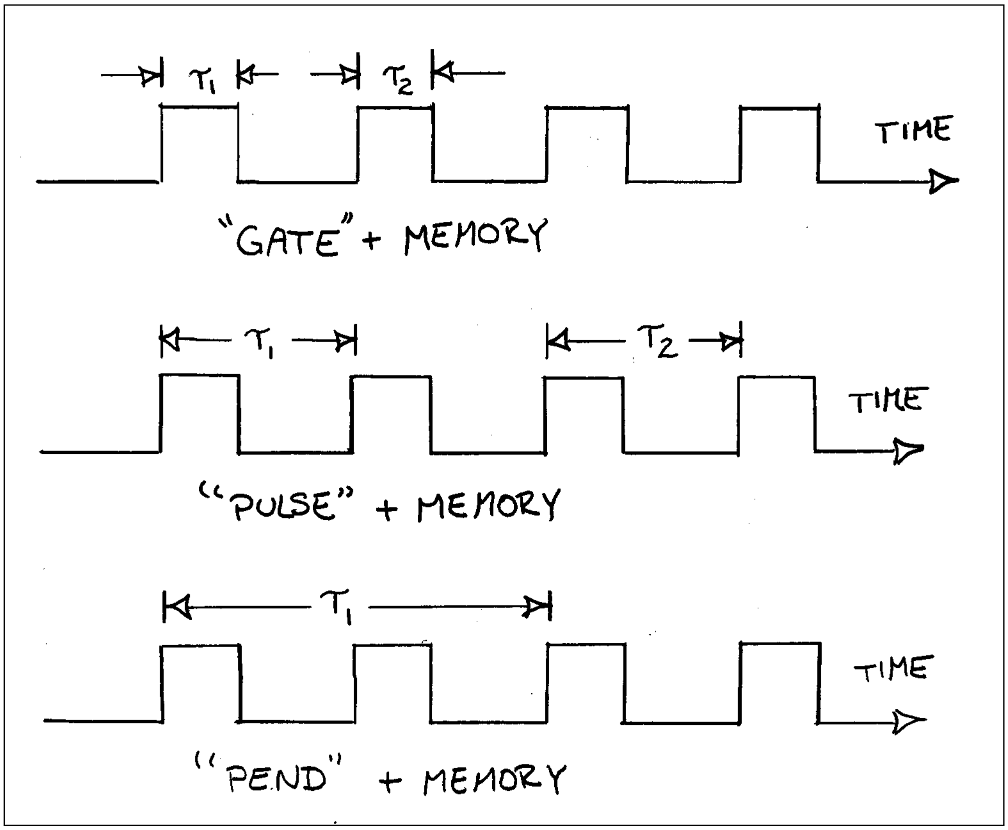

Figure 3. Response of the photogate timer to the three settings of the mode switch. The (+) sense signifies blocking of the beam.

MEASURING INSTANTANEOUS VELOCITY

We begin by studying the motion of the glider on a level, nearly frictionless air track. With the air turned on, **level the track by adjusting the leveling screw** until the glider remains stationary.

As explained in the Background section, an object's instantaneous velocity v at some point is equivalent to its average velocity measured over a vanishingly small time interval:

$$v \equiv \lim \frac{\Delta x}{\Delta t} \qquad (17)$$

We can make a good approximation to this definition by measuring the time τ that it takes for one of the flags to pass the photogate. To do this, with the calipers first **measure the width of the two flags** (they should be very nearly equal). Now, it turns out that the effective width of the flag is slightly different from than the actual width, because of the finite width of the photogate beam. The correction in mm for each photogate timer is written on the timer; a negative number means that the effective width of the flag is smaller than the physical width, and *vice versa*. You will now be able to measure the glider's velocity when you give it a push along the track.

Experiment 1a – *Instantaneous velocities on a level track*

In this and all subsequent experiments with the air track, the photogate timer is placed along the track and its height adjusted such that only the two glider flags trigger the timer. You will be giving the glider a light push towards the photogate and catching the glider after it passes the timer; otherwise, it may bounce off the end of the track and back through the timer and trigger it again before you have a chance to read your result.

The reading of the timer is the time τ that it takes for the first flag to pass through the photogate. The distance covered by the glider during this time interval is the flag's effective width d that you measured before.

The time τ and the displacement d are both short enough that their ratio gives a good approximation to the glider's instantaneous velocity at the moment the first flag passed the photogate:

$$v = \frac{d}{\tau} \tag{18}$$

You can use the timer's **MEMORY** function to measure the glider's instantaneous velocity at two different points in time. This will allow you to determine whether its velocity is changing.

1) Set the timer to **GATE** mode, and if the timer has a resolution switch (just above the reset button), set the resolution set to 0.1 ms (milliseconds). Make sure the **MEMORY** switch on the timer is set to **ON**.
2) **RESET** the timer, and give the glider a gentle push toward the timer.
3) The time τ_1 will appear on the timer. Record this value.
4) Now, to find τ_2, the amount of time that the second flag blocked the timer, flip the **MEMORY** switch to **READ**. As described in the apparatus section, the number which appears is $\tau_1 + \tau_2$. Subtract τ_1 from this number to find τ_2.
5) Now that you know τ_1 and τ_2 , you can find v_1 and v_2, the respective velocities of the glider:

$$v_1 = \frac{d}{\tau_1} \qquad v_2 = \frac{d}{\tau_2} \tag{19}$$

6) Repeat Steps 2 – 5 at least three times to get an idea of the repeatability of your experiment. If your air track is working well, the glider should move with nearly constant velocity. Do your measurements of v_1 and v_2 support this assertion?

Experiment 1b – *Instantaneous velocities on a slanted track*

1) Now raise the left end of the air track a few centimeters by placing a wooden block under the leveling screw on the left.

2) Set up a photogate timer just as before to measure the velocities v_1 and v_2 as the glider's two flags pass the beam (you may need to adjust the height of the timer so it triggers correctly).
3) This time instead of giving the glider a push, simply release it from rest a short distance above the timer, and let it slide downhill through the photogate.[2] Catch the glider so it won't bounce back through the gate again.
4) As you watched the glider, did it seem to be moving with a constant velocity, or was the velocity changing as it moved along the track?
5) Take the readings τ_1 and τ_2 from the timer as before, and use them to calculate v_1 and v_2. Repeat these measurements of τ_1 and τ_2 and calculate the corresponding v_1 and v_2 at least three times as well. Do these results indicate that the glider was moving with constant velocity, speeding up, or slowing down? Does the data support the evidence of your senses?

MEASURING ACCELERATION

If all went well, you should have found that as a glider slides downhill along an air track, its velocity increases. To quantify this change, we need to measure the glider's acceleration. Acceleration is defined as the rate of change of the velocity.

$$a = \frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \tag{20}$$

By measuring v_1 and v_2 as you have already learned to do, you can calculate the amount that the glider's velocity changed between the two measurements. If we could also find the time which passed between the measurements, $t_2 - t_1$,[3] then we could calculate the glider's average acceleration during the two measurements. Unfortunately, the photogate timer does not record the time interval between the two measurements; it only records the two separate time intervals (τ_1 and τ_2) during which the light beam was blocked by the two flags. Therefore, the best we can do is estimate the time interval from our knowledge of the distance between the flags (D), and the average velocity of the glider ($\bar{v}$):

$$t_2 - t_1 \approx \frac{D}{\bar{v}} = \frac{D}{(v_1 + v_2)/2} = \frac{2D}{v_1 + v_2} \tag{21}$$

This turns out to be a very good approximation unless something violent happens to the glider between t_1 and t_2, so we will accept it as a good approximation for the time interval. Our value of the acceleration then becomes:

$$a \approx \frac{v_2 - v_1}{t_2 - t_1} = (v_2 - v_1)\frac{(v_2 + v_1)}{2D} = \frac{v_2^2 - v_1^2}{2D}\ . \tag{22}$$

[2] If you release it more than about 30 cm above the timer, it may move so quickly that friction must then be taken into account.

[3] Note that we do *not* know the values of t_1 and t_2 – these are *not* the same values as τ_1 and τ_2.

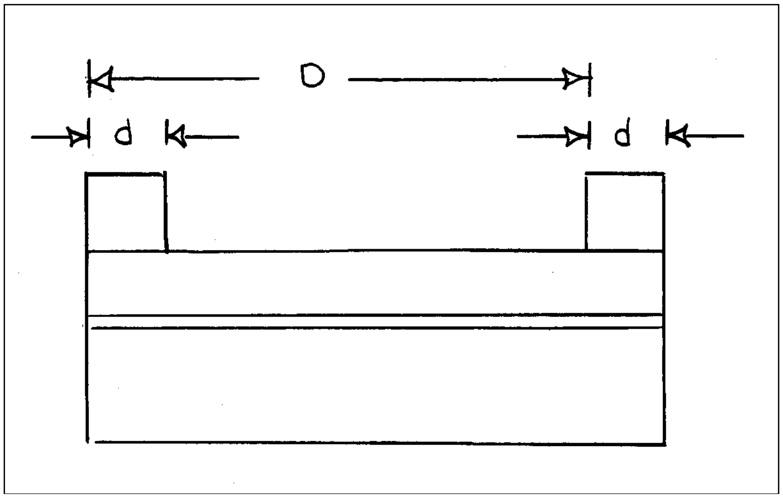

Figure 4. The definitions of distances (*D* and *d*) on the glider.

Experiment 2 – *Acceleration on a slanted track*:

You now have all of the tools you need to study accelerated motion. The procedure is as follows:

1) If you have not already done so, measure the distance *D* on your glider as shown in Figure 4.
2) Reset the timer and **release the glider from rest a short distance above the photogate.** Measure the time intervals τ_1 and τ_2, and calculate the acceleration using the above formula. Repeat this measurement several times to get a feel for the repeatability of the experiment.
3) Now, for the same tilt angle of the air track, **release the glider from a different distance from the photogate**, and for that distance measure the acceleration several times.
4) Finally, measure the acceleration for at least **three more starting points**.
5) From your experimental results, what conclusions do you draw?

KINEMATIC RELATIONS FOR CONSTANT ACCELERATION

As mentioned in the background section, there are several highly useful equations which describe motion under constant acceleration. Perhaps the most useful equation is the one giving the object's exact displacement *x* from its starting point at any time *t*:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad (23)$$

Experiment 3 – *Testing the kinematic relation*

You can test the above equation by taking data, plotting it, and using a best-fit line to find the slope.

1) Set the photogate to **PULSE** mode. If the timer has a resolution switch (just above the reset button), be sure to set the resolution to 1 ms; otherwise the counter will overflow.
2) Release the glider a known distance *x* above the photogate, as you did in the previous experiment. At the instant you release the glider, simultaneously start the photogate timer by hand using the **START/STOP** button. The timer will then record the elapsed time between the start pulse and the moment that the flag interrupts the timer beam.
3) Repeat this measurement for five or six starting points.

If the glider starts at the origin from rest and moves with constant acceleration, we may say that $x_0 = 0$, $v_0 = 0$, and a is a constant. Therefore, the motion should be described by the equation

$$x = \frac{1}{2}at^2 \tag{24}$$

4) To see if this equation is satisfied, graph your data, using x as the vertical axis and t^2 as the horizontal axis. The graph should appear as a straight line, **passing through the origin**, with slope $a/2$.
5) Draw a best-fit straight line through your data points (again, be sure that this line passes through the origin). Do most of the points lie close to the line? If so, then your data describes motion with constant acceleration.
6) Calculate the slope of your best-fit line. Twice the slope should equal the glider's acceleration. Does its value match well with the instantaneous accelerations that you measured in the previous section?

PRELAB QUESTIONS

A glider on an air track moves in the +x direction with a constant acceleration. It has two flags, each exactly 0.025 m long, with the midpoints of the flags separated by 0.200 m. The first flag interrupts the photogate timer for a time 0.056 s, and the second flag interrupts the photogate timer for a time 0.045 s.

1. What was the average velocity of the glider during the interval when the first flag was interrupted?
2. What was the average velocity of the glider during the interval when the second flag was interrupted?
3. Approximately how much time elapsed between the passage of the first flag and the passage of the second flag?
4. Calculate the approximate acceleration of the glider.

DYNAMICS

INTRODUCTION

In last week's lab you followed Galileo in investigating the key concepts of kinematics. While we learned to describe an object's motion quite elegantly, we didn't say much about the causes of the motion. It was Isaac Newton, working a generation after Galileo and building on his insights, who launched the modern science of *dynamics*, the study of forces and the ways in which they affect an object's motion through space and time.

The goal of this week's lab is to learn to use the laws of dynamics to identify the forces acting on a body and to predict its motion based on these forces. You will continue to use an experimental setup similar to Galileo's but you will be focusing on understanding the reasons for your glider's motion rather than simply recording and describing it in the language of kinematics.

A common misconception is that objects move only when some force is applied to them to "keep them going." In fact, this notion was established doctrine for centuries until Galileo realized that a moving object, which is truly free of all forces, including friction and air resistance, would not come to a stop as Aristotle maintained. It would in fact continue moving forward forever or until some force acted to stop it. It does not take any force to keep something moving with constant velocity; instead, a net force is required to *change* an object's velocity in any way. In the language of modern kinematics, a force is required to make an object accelerate (positively or negatively).

A generation later, Isaac Newton put Galileo's insight into a more precise mathematical form with his three laws of motion. Newton's first law simply repeated Galileo's observation that in the absence of all external forces, an object will remain at rest, and an object in motion will continue to move in a straight line at constant velocity forever. Newton's second law, one of the most important statements in the history of science, relates an object's acceleration to the forces acting on it:

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \tag{25}$$

In words, the instantaneous acceleration for any object is equal, in magnitude and direction, to the vector sum of all forces acting on it at that moment, divided by the object's mass.

CONSTANT FORCE (ATWOOD MACHINE)

You will find at your bench an elegant apparatus for testing Newton's second law, illustrated in **Figure 5**.

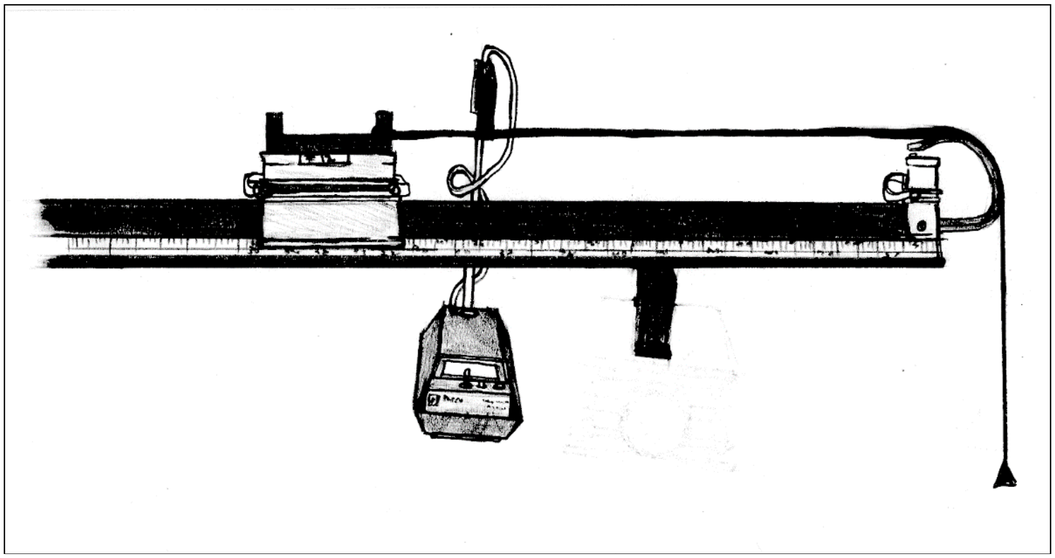

Figure 5. The Atwood machine apparatus.

The air track has a pulley on the right end, for a piece of string which connects a glider to a falling mass. The air pulley is nearly frictionless, so to a good approximation the magnitude of the force acting on the glider will just be T, the tension in the string. The downward force on the weight, on the other hand, will be $m_w g - T$ because, in addition to T pulling upward, gravity is pulling down on the weight. Since the glider and the weight are connected by the string, they will each have the same acceleration a. Applying Newton's second law, $F = ma$, to both masses, we get

$$\begin{aligned} T &= m_g a \\ m_w g - T &= m_w a \end{aligned} \tag{26}$$

If we add these two equations, we eliminate T and get a simple prediction for the acceleration of both masses:

$$\begin{aligned} m_w g &= m_g a + m_w a \\ \Rightarrow a &= \frac{m_w}{m_g + m_w} g \end{aligned} \tag{27}$$

Next, we reiterate the consequences of a mass with a constant acceleration:

$$\begin{aligned} v_x &= v_x(0) + at \\ x &= x(0) + v_x(0)t + \frac{1}{2}at^2 \end{aligned} \tag{28}$$

In the experiments described below, we will release the glider at $t = 0$. Since we will release the glider from rest, $v_x(0) = 0$. We will define the position of the glider to be the location of the leading edge of the glider. Furthermore, we will define the origin of our coordinate system to be the position of the leading edge of the glider when the glider is released from rest. With this definition, $x(0) = 0$. Our equations conveniently simplify to

$$v_x = at$$
$$x = \frac{1}{2}at^2 \tag{29}$$

Experiment 1a – Velocities and acceleration within the Atwood machine

With the air track turned on, carefully level the track until the glider has no tendency to glide one way or the other when released from rest. Now, set up the falling weight as in Figure 5. Note that the glider mass is m_g = 300.0 grams, and that the mass of the weight is m_w = 25.0 grams. Perform the following experiments and analysis:

1) Set the photogate timer in **GATE** mode, and switch the memory function **ON**.[4]
2) Releasing the glider from a point 100 mm from the photogate, and using the **MEMORY** function, measure the times τ_1 and τ_2 that the first flag and the second flag interrupt the light beam.
3) Using the formulas that we derived in the Kinematics lab, repeated below[5],

$$v_1 = \frac{d}{\tau_1} \text{ and } v_2 = \frac{d}{\tau_2}$$
$$a = \frac{v_2^2 - v_1^2}{2D} \tag{30}$$

   Calculate:
    a. The velocity, v_1, of the glider when the first flag interrupts the timer.
    b. The velocity, v_2, of the glider when the second flag interrupts the timer.
    c. The average acceleration, a, of the glider.
4) Compare this value of a with the value predicted by Equation (27).
5) Compare this value of v_1 at the first flag with the value predicted using the formula derived in the previous laboratory (Equation (16)) for a glider starting from rest, *i.e.* $v = (2ax)^{1/2}$, where $x = 100$ mm + $d/2$. Use the value of a predicted by Equation (27).
6) Do the same for v_2 of the second flag, recalling that the glider has now traveled an additional distance D.
7) Repeat this experiment with a starting point of 200 mm instead of 100 mm. In principle, you should obtain the same value of the acceleration because the acceleration is constant. How well does this obtain?

Experiment 1b – *Testing glider travel time in the Atwood machine*

1) Set the photogate timer in **PULSE** mode, and keep the **MEMORY** function on.[6]

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[4] If the timer has a resolution switch (just above the reset button), be sure to set it to 0.1 ms.
[5] Remember to correct the physical flag size to account for the calibration of the photogate timer.
[6] For this measurement, if the timer has a resolution switch (just above the reset button), set the timer resolution to 1 ms. Otherwise the register may overflow.

2) Starting the timer by hand, release the glider from 100 mm from the photogate, and measure the time that it takes for the first flag to reach the photogate timer. (The timer will start when you push the **START/STOP** button; it will stop when the leading edge of the flag interrupts the beam).
3) From the relation $x = (1/2)at^2$, your knowledge of x (=100 mm), and of your predicted value of a given by Equation (27), predict the time that it takes for the first flag to reach the photogate. Does the time reasonably agree with your measured value?
4) Repeat this experiment with a starting point of 200 mm instead of 100 mm. Does your predicted time for this new x still reasonably agree with your measured value?

CONSTANT FORCE (SLOPING AIR TRACK)

In today's second set of experiments, we will repeat Galileo's classic experiment: an object slides without friction down a track inclined at an angle θ with respect to the horizontal, as in **Figure 6**. In last week's labs we used this as a test bed for kinematics; this week we will investigate the dynamics of the motion.

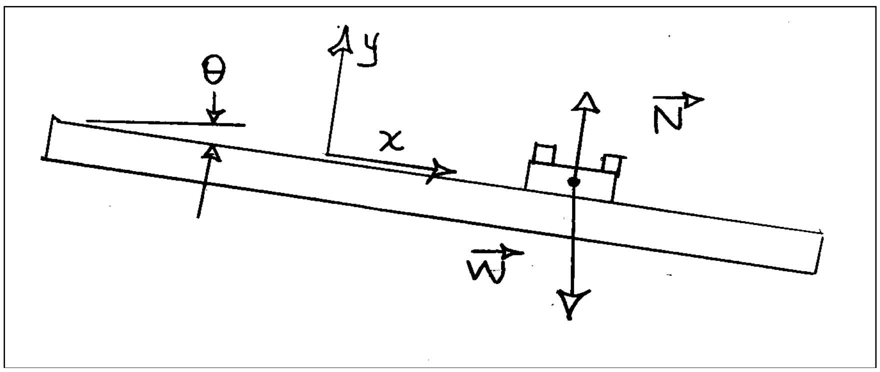

Figure 6. The coordinate system of the tilted air track.

While natural laws are most elegantly expressed in vector notation as in Equation (25), most practical calculations are easier when the vectors are broken into components. In component form, the above equation is actually three equations, one for each Cartesian direction *in the frame of the air track*:

$$a_x = \frac{\sum F_x}{m} \quad a_y = \frac{\sum F_y}{m} \quad a_z = \frac{\sum F_z}{m} \tag{31}$$

We recognize that there are just two forces are acting on the glider: gravity, arising from the attraction between the glider and the earth; and the *normal* force, which is the force of the cushion of air pushing on the glider in the direction perpendicular to the air track.[7]

[7] In traditional geometry textbooks, a line that is perpendicular to another line is said to be *normal* to that line.

The force of gravity on the glider is directed toward the center of the earth, and is numerically equal to the weight. It has the following three components in the frame of the air track:

$$\begin{aligned} W_x &= mg\sin\theta \\ W_y &= -mg\cos\theta \\ W_z &= 0 \end{aligned} \tag{32}$$

where we have oriented our coordinate axes with the x-axis parallel to the air track, and the y-axis perpendicular to the air track.

If *N* is the *magnitude* of the normal force, normal force components are given by

$$\begin{aligned} N_x &= 0 \\ N_y &= N \\ N_z &= 0 \end{aligned} \tag{33}$$

By vectorially adding the two forces on the glider, our three components of Newton's law become

$$\begin{aligned} a_x &= \frac{F_x}{m} = \frac{W_x + N_x}{m} = \frac{mg\sin\theta + 0}{m} = g\sin\theta \\ a_y &= \frac{F_y}{m} = \frac{W_y + N_y}{m} = \frac{-mg\cos\theta + N}{m} = -g\cos\theta + \frac{N}{m} \\ a_z &= \frac{F_z}{m} = \frac{W_z + N_z}{m} = \frac{0+0}{m} = 0 \end{aligned} \tag{34}$$

These three equations tell us three separate stories:

The first equation tells us that the glider moves along the air track with an acceleration that depends upon the angle θ of the ramp, but not on the mass of the glider. As a reality check, note that the acceleration is zero when the air track is horizontal; and the acceleration is g when the air track is vertical (please don't try this!).

The second equation involves the hitherto-mysterious normal force N. Since the y-component of acceleration is zero (because $y = 0$ always), we get $N = mg\cos\theta$. As a reality check, note that the normal force is just the weight of the glider when the air track is level; and the normal force would be zero if the air track were vertical.

The third equation tells us what we knew all along; that the glider does not move sideways; i.e. $z = 0$ always.

Experiment 2a - Velocities and acceleration with a sloping air track

Remove the string and the hanging mass that you used in the previous experiment. With the air track turned on, carefully level the track until the glider has no tendency to glide one way or the other when released from rest.

1) Now, elevate the left end of the track and measure the elevation difference *h* between the two supports. Using basic trigonometry, find the angle that the track makes with the horizontal.[8]

2) Repeat the measurements that you made with the Atwood machine in **Experiment 1a**, using for your predicted acceleration $a_x = g\sin\theta$.

Experiment 2b - *Testing glider travel time with a sloping air track*

1) Repeat the measurements that you made with the Atwood machine in **Experiment 1b**, using for your predicted acceleration $a_x = g\sin\theta$.

PRELAB QUESTIONS

1. For the Atwood machine apparatus, draw two free-body diagrams which clearly indicate the forces acting on:
   i. The glider
   ii. The hanging mass

For the following problems, use a glider mass of 0.420 kg.

2. Atwood machine: A glider moves frictionlessly along a level air track, tugged by a vertically suspended mass of 0.075 kg that is attached to the glider with a string that is looped over the pulley (refer Figure 5). What is the expected acceleration of the glider?
3. Sloping air track: (Assume the vertically suspended mass in problem 2 has been removed). A glider moves frictionlessly along an air track that has been tilted by 1.5 inches over a span of 1.30 meters. What is the expected acceleration of the glider?

[8] Make sure that the angle is less than about 10 degrees, so that friction will not be a serious consideration.

GRAVITATIONAL, KINETIC, AND ROTATIONAL ENERGY

INTRODUCTION

You have learned from the lectures and the textbook that energy is one of the most important and powerful concepts in physics, because the total energy in a closed system cannot change. However, using this law requires that we have a truly closed system, so that energy can neither enter nor escape; and it requires that we identify all possible forms of energy.

In this laboratory we will explore the consequences of the law of conservation of energy under special, limited circumstances.[9] In particular, we will consider the energy of a so-called conservative system, where the only forms of energy that we need to consider are potential energy and kinetic energy. The law of conservation of energy in this limited case can be stated very simply:

> There exist closed mechanical systems, which we call *conservative* systems, where the sum of the potential energies and kinetic energies of all the particles is constant in time (conserved).

As an example, our solar system of the sun and eight planets can be regarded as a conservative system to very high accuracy.

In this laboratory, we will further restrict our discussion to so-called *rigid* bodies, where the particles are aggregated into solids whose shape does not change. In such cases, the total kinetic energy of a body, which is the sum total of the kinetic energies of the constituent particles, is simply the sum of two numbers: the total *translational* kinetic energy of the body, and the total *angular* kinetic energy of the body.

Such a system can be regarded as *conservative* if we can neglect the transfer of energy to and from the bodies to the outside world. Examples of such non-conservative transfers include heat transfer, air resistance, friction, sound or light waves, *etc.* We have endeavored to keep these effects to a minimum in the following laboratory.

THE PENDULUM

Consider the motion of a pendulum. Upon releasing the bob, it swings back and forth, returning again and again to a location very near the point of first release. Indeed, it seems reasonable to assume that in the case of a perfect pendulum, the bob would oscillate forever this way. This suggests the operation of a conservation principle: although the motion of the bob is not uniform, something within the system is being held constant (is being conserved), so that the bob always returns to the same position. We know from the textbook that the conserved quantity is the total energy, which is the sum of the kinetic energy of the bob and the potential energy of the bob.

[9] In next week's lab, we will explore in greater detail the laws of conservation of momentum and of energy.

Now the kinetic energy of the bob is just $K = (1/2)mv^2$, and the potential energy is just $U = mgh$, where h is the height of the bob relative to any fixed horizontal reference plane.[10] Thus, our conservation law for the speed and the height at any two arbitrary times t_1 and t_2 is simply (where g =9.8 m/s^2 is the acceleration of gravity)

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ \frac{1}{2}mv_1^2 + mgy_1 &= \frac{1}{2}mv_2^2 + mgy_2 \\ \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 &= mg(y_2 - y_1) \end{aligned} \tag{35}$$

$$v_1^2 - v_2^2 = 2g(y_2 - y_1) \tag{36}$$

If the bob is released at rest, then $v_1 = 0$; and if the height of the bob when released (relative to its lowest position) is h, then $y_2 - y_1 = -h$, so

$$\boxed{v_2^2 = 2gh} \tag{37}$$

Using the substitution for h as discussed in the prelab, v_2 can be rewritten as:

$$v_2 = 2\sqrt{gl}\,\sin(\theta/2) \tag{38}$$

A great advantage of this approach is that nowhere do we have to even think about forces. In fact, in more advanced courses, the concept of force is abandoned altogether, in favor of the concepts of energy and momentum.

Notice also that the above result shows that the speed of the pendulum bob (once we are given the height and g, the acceleration of gravity) does not depend upon the mass of the bob, nor does it depend upon the length of the string[11]!

At your station you will find a simple pendulum made from a cylinder of various materials, suspended from a hook by a string. You will also find a photogate timer. (Refer to **Figure 7**). The mass of the brass bob is 69.7 grams, and the mass of the aluminum bob is 24.1 grams.

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[10] Remember, we can always define the zero of potential energy in any way that we like, as long as we keep that definition throughout.

[11] Although an l is present in the Equation (38), remember that we are only using it as a convenient way to calculate the height h. We could just as well measure h directly from the bob's lowest position with a ruler, thus removing l from the equation.

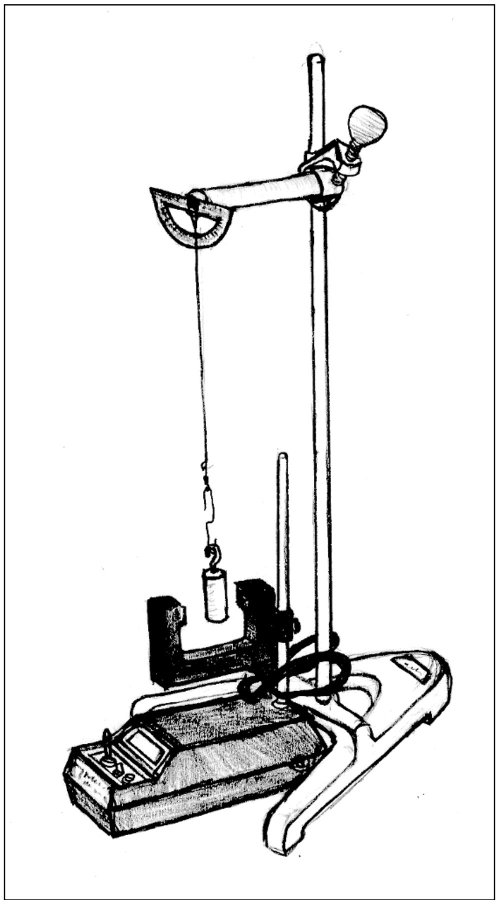

Figure 7. The pendulum and photogate timer.

Experiment 1a – *Finding* g *using the pendulum velocities*

1) Set the photogate timer in **GATE** mode. If your timer has a resolution switch, select the fine (0.1 ms) scale.
2) Determine the height of the bob relative to its equilibrium position, using the length of the pendulum and the protractor provided (refer to the prelab problem).
3) Now, draw the bob back and release it from rest ($v_1 = 0$), taking care to measure the angle of the bob upon releasing it.
4) Using the photogate timer as shown in Figure 7, measure the speed of the bob at the bottom of its travel (v_2) for at least four starting angles. (Don't forget that the effective diameter of the bob is slightly different from the true diameter because of the photogate correction).
5) Make a graph of v_2 versus $\sin(\theta/2)$. Using the results of prelab question #1, you should get a straight line with slope $2\sqrt{gl}$. Measure the slope of the line, and make a determination of g.
6) Repeat the above with the aluminum mass.

Experiment 1b – *Potential and kinetic energy of the pendulum*

1) For at least three starting angles, calculate the potential energy of the brass bob at the release point, and compare it with the kinetic energy at the bottom of its travel.

ENERGY CONSERVATION FOR A ROLLING BALL

Now that you are comfortable with the relationship between kinetic and potential energy, and the conservation of the sum of the two, you can explore the behavior of a ball rolling down a ramp. At your lab station is a set of four metal ramps, and some metal spheres which you will allow to gain kinetic energy by rolling down the ramp. We will use the principle of conservation of energy to determine how far the ball will travel horizontally before it hits the floor (the distance d_2 in Figure **8**).

Sliding motion

We first consider a hypothetical situation where the ball *slides* down a frictionless ramp, such as an air track (we won't actually be doing this experiment, but it is nevertheless useful to consider this simpler situation). Since the ramp is frictionless, and since we are neglecting air resistance, we can say that the total energy of the ball (kinetic plus potential) is constant. Mathematically, this is expressed (our convention is that positive y is the up-wards direction) as

$$\frac{1}{2}Mv_1^2 + Mgy_1 = \frac{1}{2}Mv_2^2 + Mgy_2 \qquad (39)$$

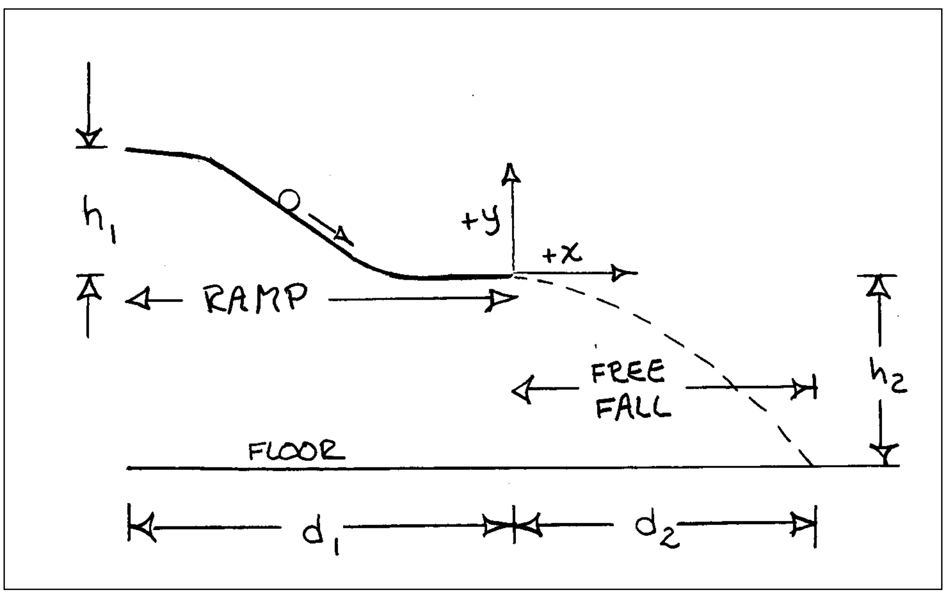

Figure 8. The ramp coordinate system.

Now, in all of our experiments, we will be releasing the ball from rest, so that $v_1 = 0$. Also, since the height of the ramp is defined as h_1, we have y_1-y_2 = h_1. Therefore, we can derive a simple expression for the square of the velocity at the bottom of the ramp:

$$v_2^2 = 2g(y_1 - y_2) = 2gh_1$$
$$\Rightarrow v_2 = \sqrt{2gh_1} \tag{40}$$

At the bottom of the ramp, the ball will fly off horizontally and head to the floor. (For consistency we will always use the green ramp). Our job is to figure out how far the ball travels horizontally before it hits the floor. Now, we know from the textbook that we can write the *x* and *y* components of the motion separately, in the following form:

$$x(t) = x(0) + v_x(0)t$$
$$y(t) = y(0) + v_y(0)t - \frac{1}{2}gt^2 \tag{41}$$

For this part of the ball's trip, we can take $t = 0$ to be the moment that the ball leaves the ramp. Also, we can take as the origin of our coordinate system the position of the ball at the end of the ramp. With these conventions, $x(0) = y(0) = 0$.

Since the ramp is horizontal at the very end, we can also say that $v_y(0) = 0$, and that $v_x(0) = v_2$, as calculated in Equation (40). Then, Equations (41) become

$$x(t) = v_2 t = \sqrt{2gh_1}\,t$$
$$y(t) = -\frac{1}{2}gt^2 \tag{42}$$

We can eliminate *t* from these equations, and write *x* in terms of *y*:[12]

$$x(y) = v_2\sqrt{\frac{-2y}{g}} = \sqrt{-4h_1 y} \tag{43}$$

Finally, we know that the ball has dropped a distance h_2, so that we can say at the floor $y = -h_2$. So our final result is

$$\boxed{x(y = -h_2) = d_2 = \sqrt{4h_1h_2} = 2\sqrt{h_1h_2}} \text{ (sliding object)} \tag{44}$$

This is a very simple and elegant result. If an object slides down a frictionless ramp of total height h_1, is then launched horizontally, and then falls a vertical distance h_2, it will travel a horizontal distance equal to twice the geometric mean of h_1 and h_2. Notice that this result does not depend upon the acceleration of gravity, *g*, nor upon the mass of the object *M*!

[12] Since the object is falling from its initial position $y = 0$, and since our convention is that upward displacements are positive, the numerical value of *y* will be negative after $t = 0$.

Experiment 2a – Examining the effect of mass on x

In all of these experiments we will use the ramp with the green dot at the end (this ramp has the least curvature of the four ramps). First, place the wooden landing platform on the floor parallel to the path of the ball, and use the plumb bob to accurately position the "0" mark on the tape measure vertically below the end of the ramp. After trial runs to help you get things set up, tape a 1-foot piece of cash register tape to the landing platform in the region where the ball will fall.

1) Release the solid steel sphere (the heaviest one) from the top of the ramp, being careful that it starts from rest after you remove your hand.
2) When the ball hits the register tape, it will leave a round mark that you can use to measure the horizontal distance *x* traveled by the ball as it falls to the floor. Repeat the measurement several times to get a feel for the accuracy of the result.
3) Now repeat the above experiment with the solid aluminum sphere. Aluminum is much lighter, so this experiment will test the prediction that the horizontal distance is independent of the mass of the sphere.
4) Compare the horizontal distance traveled by each ball with the prediction from Equation (44).

You may be disappointed to observe that your observed horizontal distance for both the steel and aluminum spheres is significantly different from the value that you predicted in Equation (44). This result, on the face of it, suggests that there is something definitely wrong, or at least incomplete, with our analysis.

To make a long story short, it turns out that we failed to include *rotational* kinetic energy in our analysis. It is easy to see that a spinning object possesses a significant amount of kinetic energy. Even if an object is otherwise stationary, individual parts of it may be moving rapidly as they spin about the rotational axis. If we drop a spinning object (a tire, a coin, etc.) when it hits the ground it will start moving forward, converting some of its rotational kinetic energy to translational (motional) kinetic energy.

Just as the translational kinetic energy of an object is given by $\frac{1}{2} mv^2$, the rotational kinetic energy is $\frac{1}{2}I\omega^2$, where I is the so-called *moment of inertia*, and ω is the object's angular velocity $d\theta / dt$. The moment of inertia depends upon the body's mass, but it also depends upon the *distribution* of mass about the center of rotation. So we need to include a term in our energy Equation (39) to account for rotational kinetic energy. When we do so, we find that our final result will depend upon the exact distribution of matter. The results, which are proved in the appendix, are summarized as follows:

$$x(y=-h_2)=d_2=2\sqrt{h_1h_2} \text{ (sliding object)} \quad (45)$$

$$x(y=-h_2)=d_2=2\sqrt{5/7}\sqrt{h_1h_2}=1.690\sqrt{h_1h_2} \text{ (rolling solid sphere)} \quad (46)$$

$$x(y=-h_2)=d_2=2\sqrt{3/5}\sqrt{h_1h_2}=1.549\sqrt{h_1h_2} \text{ (rolling hollow sphere)} \quad (47)$$

These results will be discussed in the lectures toward the end of the quarter, so the proof is beyond the scope of the course thus far.[13] However, one can see from the formulas that the ball will travel significantly less as a consequence of the rotational kinetic energy. This happens because the total energy available to the ball from its trip down the ramp is just mgh_1; but if the ball is rolling this energy must be shared between the translational motion and the rotational motion, which in turn lessens the amount of energy available to translational motion. This effect will vary depending upon whether the sphere is solid, hollow, or has an even more complicated distribution of mass.

Experiment 2b – *Examining the effect of mass distribution on x*

1) Using the same methods as in **Experiment 2a**, measure the horizontal distance traveled by a steel ball that is actually hollow.
2) Compare quantitatively the experimental values of x for each of the three balls to the value predicted using the sliding equation.
3) Compare quantitatively the experimental values of x for each ball to the appropriate theoretical value (Equations (46) and (47)). Comment on the consequence of the amount of mass (solid steel vs. solid aluminum) versus the consequence of the distribution of mass (hollow steel or solid steel).

APPENDIX: ROTATIONAL MOMENTS OF INERTIA

Although rotational motion is beyond the scope of the course at this point, if you are interested in the origins of Equation (45), (46), and (47), we derive them below.

Rolling motion

Having analyzed the motion of a mass sliding frictionlessly down a ramp, we can now study the more complicated case of a sphere that *rolls* down the ramp, rather than slides. (This closely resembles the original experiments of Galileo). In this case we must include the rotational kinetic energy in our conservation formula. Now, the rotational kinetic energy of an object is given by

$$K_R = \frac{1}{2} I\omega^2 \tag{48}$$

Where K_R is the rotational kinetic energy, I is the moment of inertia about the rotational axis, and ω is the angular velocity of the object, expressed in radians per second. The moment of inertia of a solid sphere about an axis passing through the center of a sphere of radius R and mass M is given by[14]

$$I = \frac{2}{5} MR^2 \tag{49}$$

Since the object is rolling along the ramp with translational velocity v, the angular velocity is related to the speed by

[13] If you are interested in pursuing this matter further, please refer to the appendix of this section.

[14] Tables of moments of inertia for various shapes can be found in the Handbook of Chemistry and Physics.

$$\omega = \frac{v}{R} \tag{50}$$

Putting these last two formulas together, we have the rotational kinetic energy of a homogeneous sphere to be

$$K_R = \frac{1}{2}(\frac{2}{5}MR^2)\left[\frac{v}{R}\right]^2 = \frac{1}{5}Mv^2 \tag{51}$$

Now we can update our conservation of energy formula to include rotational kinetic energy:

$$\frac{1}{2}Mv_1^2 + \frac{1}{5}Mv_1^2 + Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{1}{5}Mv_2^2 + Mgy_2 \tag{52}$$

It is now easy to repeat the earlier (sliding-sphere) steps for the rolling case, with this slightly modified kinetic energy. The intermediate steps are as follows:

$$\begin{aligned} v_2^2 &= \frac{10}{7}g(y_1 - y_2) = \frac{10}{7}gh_1 \\ \Rightarrow v_2 &= \sqrt{\frac{10}{7}gh_1} \end{aligned} \tag{53}$$

$$x(y) = v_2\sqrt{\frac{-2y}{g}} = \sqrt{-\frac{20}{7}h_1 y} \tag{54}$$

$$\boxed{x(y=-h_2) = d_2 = \sqrt{\frac{20}{7}h_1 h_2} = 1.690\sqrt{h_1 h_2} \text{ (rolling solid sphere)}} \tag{55}$$

Again, this is a very simple and beautiful result. Like the sliding sphere result, it states that the horizontal distance is independent of the acceleration of gravity and of the mass of the object. Moreover, it states that the rolling solid sphere will not travel as far horizontally as the sliding sphere. This is because the total potential energy available for conversion, Mgh_1, is the same in both cases, but for the rolling sphere, the final speed at the bottom of the ramp is less, because the potential energy is now shared between the translational kinetic energy and the rotational kinetic energy.

Spherical shell

Our third and final case study will be a metal sphere that has been hollowed out (a thin spherical shell). It turns out that there is again a very simple expression for the moment of inertia:

$$I = \frac{2}{3}MR^2 \text{ (spherical shell)} \tag{56}$$

We can now repeat all of the above steps, but with this new moment of inertia. Basically, we just need to replace the factor of (2/5) in Equation (51) with a factor of (2/3). The final result is

$$\boxed{x(y=-h_2)=d_2=\sqrt{\frac{12}{5}h_1h_2}=1.549\sqrt{h_1h_2}\ \text{(rolling spherical shell)}} \qquad (57)$$

Interestingly, the spherical shell will travel an even shorter distance than the solid sphere. This is because a larger proportion of the potential energy, Mgh_1, is converted to rotational kinetic energy. Also, notice that the result does not depend upon the thickness of the shell[15].

PRELAB QUESTIONS

1. (Pendulum section) Suppose a pendulum bob is suspended from a pivot by a mass-less string of length l, measured to the center of the bob.
    a. Show that if I draw the pendulum back by an angle θ, the center of the bob is elevated by the distance:

$$\Delta h = l(1-\cos\theta) = 2l\sin^2(\theta/2) \qquad (58)$$

    b. Using the results of part (a), show that the speed of the pendulum bob at the bottom of its travel should be:

$$v = 2\sqrt{gl}\,\sin(\theta/2) \qquad (59)$$

Hint: recall that $(1-\cos(\theta))/2 \equiv \sin^2(\theta/2)$.

2. (Ramp section) Suppose the height of the ramp is h_1 = 0.45 m, and the foot of the ramp is horizontal, and is h_2 = 1.6 m above the floor. What will be the horizontal distance traveled by the following four objects before they hit the floor? Assume that R = 12 mm in each case; assume that the density of steel is 7.8 g/cm^3; and assume that the density of aluminum is 2.7 g/cm^3.
    a. A solid steel sphere *sliding* down the ramp without friction.
    b. A solid steel sphere *rolling* down the ramp without slipping.
    c. A spherical steel shell with shell thickness 1.0 mm *rolling* down the ramp without slipping.
    d. A solid aluminum sphere *rolling* down the ramp without slipping.

[15] This is specifically for our case of a thin-shelled sphere; if the shell had significant thickness, its moment of inertia would be different.

MOMENTUM AND ENERGY CONSERVATION

CONSERVATION OF MOMENTUM

To understand the law of conservation of momentum, it is helpful to imagine that the universe is entirely comprised of many atoms, each possessing a positive mass m_i, where the index i runs over all the atoms in the universe. At any moment in time t, each atom is moving with a vector velocity $\mathbf{v}_i$(t). For each atom i, the momentum at time t is a vector given by the familiar expression

$$\vec{\mathbf{p}}_{\mathbf{i}}(t) \equiv m_i \vec{\mathbf{v}}_{\mathbf{i}}(t) \qquad (60)$$

We now define the total momentum of a collection of atoms labeled m through n to be just the *sum* of the individual momenta:

$$\vec{\mathbf{P}}(t) \equiv \sum_{i=m}^{i=n} \vec{\mathbf{p}}_{\mathbf{i}}(t) \qquad (61)$$

The law of the conservation of momentum is very simple. It states the following:

If a group of atoms, say m through n, interact only among themselves, but not with any other atoms in the universe (in the time interval under consideration), we say the system is <u>closed</u>. In such a closed system, the three components of the total linear momentum are each constant in time.

Mathematically, this is expressed as follows:

$$\vec{\mathbf{P}}(t) = \text{constant} \qquad (62)$$

To this day no known exception to this law has been observed, even in the quantum domain, and even in the relativistic domain (where the definitions of momentum are generalized versions of Equation (60)).

At this point it is useful to say a bit more about "a collection of atoms." At one extreme is the so-called "rigid body," which is an idealized assembly of atoms with total mass M that all move in unison. For a rigid, non-rotating body, each atom i moves with the same velocity $\vec{\mathbf{v}}$. Therefore, the total momentum of the body is just

$$\vec{\mathbf{P}}(t) \equiv \sum_{i=m}^{i=n} \vec{\mathbf{p}}_{\mathbf{i}}(t) = \sum_{i=m}^{i=n} (m_i \vec{\mathbf{v}}) = \vec{\mathbf{v}} \sum_{i=m}^{i=n} m_i = M\vec{\mathbf{v}} \qquad (63)$$

Not surprisingly, the momentum of the rigid body is just the total mass of the body, multiplied by the vector velocity of the body.

It turns out that for a collection of mutually interacting atoms that are otherwise isolated from the rest of the world (say a solid, a liquid, a gas, or even a galaxy), there is a point in space, called the center of mass, that moves uniformly with a velocity $\vec{\mathbf{v}}$. Just as in the case of a rigid body, the total momentum of the collection is a constant, and is given by the same expression as for a rigid body:

$$\vec{\mathbf{P}}(t) \equiv \sum_{i=m}^{i=n} \vec{\mathbf{p}}_{\mathbf{i}}(t) = M\vec{\mathbf{v}} \tag{64}$$

An immediate consequence of the law of conservation of momentum is the fact that if the total momentum of a collection of atoms changes with time, then there necessarily must have been an external force acting on the collection of atoms. For example, you cannot, from rest, walk across a room without having an external force (the friction between your soles and the floor) act on you; you cannot, from rest, walk across an ice-covered pond that is perfectly slippery.

CONSERVATION OF ENERGY

As with the law of the conservation of momentum, it is helpful to consider an isolated set of atoms of mass m_i, where i runs from m through n, that do not interact at all with the outside world, but which may interact with one another. The *kinetic energy* of each atom, K_i, is defined as

$$K_i = \frac{1}{2} m_i \left|\vec{\mathbf{v}}_{\mathbf{i}}\right|^2 \tag{65}$$

The total kinetic energy is just the sum of the individual kinetic energies:

$$K = \sum_{i=m}^{i=N} K_i \tag{66}$$

In addition, each *pair* of atoms can possess a mutual potential energy, U_{ij}, as a consequence of their mutual electrostatic and gravitational interaction.[16] The total potential energy is obtained by summing the potential energy of each pair of atoms. For example, if we have four atoms, the total potential energy will be

$$\begin{aligned} U &= U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34} \\ U &= \frac{1}{2}(U_{12} + U_{21}) + \frac{1}{2}(U_{13} + U_{31}) + \frac{1}{2}(U_{14} + U_{41}) + \\ &+ \frac{1}{2}(U_{23} + U_{32}) + \frac{1}{2}(U_{24} + U_{42}) + \frac{1}{2}(U_{34} + U_{43}) \end{aligned} \tag{67}$$

[16] The potential energy of a pair of atoms does not depend upon how we label the atoms. Therefore, $U_{ij} = U_{ji}$

Thus, the total potential energy of a collection of atoms can be written in the compact summation notation as:[17]

$$U = \frac{1}{2}\sum_{i \neq j}\sum_{j} U_{ij} \tag{68}$$

Finally, the total energy of the system of atoms is defined to be the sum of the kinetic and potential energies:

$$E = K + U \tag{69}$$

The law of conservation of energy states the following: In a *conservative* system, the total potential plus kinetic energy of a collection of an isolated group of atoms is a constant (it is conserved). [18]

We have discussed the conservation of energy at the atomic level, where the law appears in its most basic form. Before people understood that matter is comprised of atoms, it was not possible to formulate the law, because the purely *mechanical* kinetic energy of an isolated set of objects is not always conserved. For example, two gliders on the air track in this lab moving with equal and opposite speeds can collide and stick together, with the pair coming to rest. The macroscopic mechanical energy of the gliders has vanished; but the internal energy of the gliders (the motions of the constituent atoms) has taken up the lost energy, which is manifested by the increase in the temperatures of the gliders. In a sense energy can be "hidden," because it is not manifested in the gross motion of the objects. Momentum, on the other hand, cannot be concealed among the constituent atoms, because it has vector (directional) properties.[19]

Remarks: The laws of conservation of momentum and energy are among the most fundamental and the most useful laws of physics. Although the preceding account is sufficient for the vast majority of situations that you will encounter, it turns out that the theory of relativity requires us to adopt somewhat more general definitions of momentum and energy, to account for motions approaching the speed of light; when we do so, momentum and energy are still conserved. Also, it turns out that the electromagnetic field itself can contain momentum and energy. This is also true for the exotic particles associated with binding of the atomic nucleus (the "weak" force and the "strong" force). These are the subjects of more advanced courses in physics, and can be ignored in most everyday situations.

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[17] Since an atom does not have a potential energy relative to itself, we assume $U_{ii} = 0$ for all values of i.

[18] Strictly speaking, we must assume that no radiant energy (light) enters or escapes the collection of atoms.

[19] You may be wondering about how *chemical* energy fits into this picture. Chemical energy is simply another manifestation of the potential energy shared by molecules, which are bound clusters of atoms. In a chemical reaction, A+B→C+D, the total energy (kinetic plus potential) is conserved, provided that we carefully keep track of the potential energies of the constituent atoms in each of the four molecules.

CONSERVATION LAWS: ELASTIC COLLISIONS

The study of collisions of particles is central to physics, chemistry, and biology. In physics, we have learned about the fundamental interaction laws between elementary particles through collisions produced in high energy particle accelerators, such as the electron accelerator at SLAC (Stanford, California), and at the proton accelerators near Chicago (Fermilab) and Geneva, Switzerland (CERN). In chemistry and biology, the collisions between atoms or the collisions between molecules lead to the formation or destruction of chemical compounds.

An important subset of such collisions is described by the so-called elastic collision process,

$$A + B \to A + B \tag{70}$$

where the sum of the kinetic energies of the projectile and the target before the collision is equal to the sum of the kinetic energies of the projectile and the target after the collision. In other words, no energy goes into the *internal* kinetic and potential energies of the objects.

In this laboratory we will take objects A and B to be the familiar gliders which can move frictionlessly along the air track. We define v_A and v_B to be the initial velocities of gliders A and B, and v'_A and v'_B to be the final velocities. The conservation laws of energy and momentum can thus be written

$$\begin{aligned} &m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \text{(momentum)} \\ &\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v'^2_A + \frac{1}{2} m_B v'^2_B \quad \text{(energy)} \end{aligned} \tag{71}$$

We will consider two cases in this lab where the target is initially at rest. Thus, we may set $v_B = 0$, resulting in the following two equations:

$$\begin{aligned} m_A v_A &= m_A v'_A + m_B v'_B \\ m_A v_A^2 &= m_A v'^2_A + m_B v'^2_B \end{aligned} \tag{72}$$

Since we have two equations in two unknowns (v'_A and v'_B), if we know the masses and the projectile's initial velocity, we should be able to solve for the unknowns. It turns out that these equations are not too hard to solve. We begin by rewriting the two equations:

$$\begin{aligned} m_B v'_B &= m_A v_A - m_A v'_A = m_A (v_A - v'_A) \\ m_B v'^2_B &= m_A v_A^2 - m_A v'^2_A = m_A (v_A - v'_A)(v_A + v'_A) \end{aligned} \tag{73}$$

We can now divide the second equation by the first, to get

$$v'_B = v_A + v'_A \quad \Rightarrow \quad v'_B - v'_A = v_A \tag{74}$$

This important and surprisingly simple intermediate result tells us that if the target is initially at rest, the target and the projectile move off with a relative velocity which is just the projectile's initial velocity!

To finish the story, all we have to do is plug this expression for v'_B into our original momentum conservation formula, Equation (72):

$$\begin{aligned} &m_A v_A = m_A v'_A + m_B (v_A + v'_A) \\ &\text{or,}\ \ (m_A - m_B) v_A = (m_A + m_B) v'_A \end{aligned} \tag{75}$$

$$\boxed{\text{or,}\ \ v'_A = \frac{m_A - m_B}{m_A + m_B} v_A \ \text{(elastic; target initially at rest)}} \tag{76}$$

Finally, we can use Equation (74) to find the corresponding result for v'_B:

$$\boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A \ \text{(elastic; target initially at rest)}} \tag{77}$$

We will consider the case when glider A collides with another glider of equal mass but which is at rest. If we take the masses to be equal ($m_A = m_B$), then Equations (76) and (77) read $v'_A = 0$ and $v'_B = v_A$. This result, which is dramatically illustrated with Newton's cradle, tells us that the projectile stops in its tracks, while the target picks up all of the momentum and energy.

Experiment 1 – *Elastic collisions*

To test the above conclusion, start by leveling the air track as best you can. Set up the projectile and the glider so that magnetic bumpers of the same sign face each other.

1) Using two photogates, send the projectile glider toward the stationary target glider, and measure both the initial speed of the projectile and the final speed of the target.[20]
2) Repeat the experiment at least three times, recording and interpreting the initial and final velocities of the projectile and of the target. How well do your results compare with the prediction from Equations (76) and (77)? Was energy conserved? Was momentum conserved?
3) Say glider A (the projectile glider) were more massive than glider B (the target glider). Qualitatively explain how their final velocities would differ from the case where the gliders have the same mass.

[20] For best results, the glider should be moving between 300 and 450 mm/s (this corresponds to a value of τ between 0.055 and 0.08 seconds)

CONSERVATION LAWS: INELASTIC COLLISIONS

We now investigate the collision of two objects, resulting in them sticking together. Unfortunately, we cannot use the law of conservation of energy, because in sticking together the object will have transferred energy to the internal motions of the constituent atoms, in the form of heat, which we cannot accurately measure in this lab. However, all is not lost; we know at least that the two objects are emerging with the same velocities, because they are stuck together. Our conservation of momentum law then tells us that

$$m_A v_A + m_B v_B = (m_A + m_B) v'_{AB} \tag{78}$$

This is even easier than the preceding experiment, because we can write down the general solution by inspection, even if the masses are not the same, and even if the target is initially moving!

$$\boxed{v'_{AB} = \frac{m_A v_A + m_B v_B}{m_A + m_B} \text{ (fully inelastic)}} \tag{79}$$

Of course, if the masses are both equal (as in this experiment) and if the target is initially at rest, we get the trivial result

$$\boxed{v'_{AB} = \frac{1}{2} v_A \text{ (fully inelastic,equal masses, target initially at rest)}} \tag{80}$$

In other words, a moving glider colliding with, and sticking to, an identical stationary glider results in the pair moving off with exactly half the initial velocity.

Experiment 2 – *Inelastic collisions*

Set up the two gliders so that magnetic bumpers of the opposite sign are facing each other. This way, when they collide, they will stick together and travel as a single glider.

1) Send the projectile glider toward the stationary target glider. Measure the initial velocity of the projectile glider, and the final velocity of the pair stuck together.

2) Repeat the experiment at least three times, recording and interpreting your results in light of the preceding analysis. How well do your results compare with the prediction from Equation (79)? Was energy conserved? Was momentum conserved?

Thus far we have studied perfectly elastic and perfectly inelastic collisions. Between the two lies the vast spectrum of partially inelastic collisions, where some, but not all, of the energy is dissipated as heat, or light, or sound, or other form of energy. In such cases, total energy and total momentum is still conserved, but we must keep careful track of all possible manifestations of momentum and energy.

PRELAB QUESTIONS

1. Two gliders of equal mass m and equal and opposite initial velocity v collide perfectly elastically. Using both the momentum and energy conservation equations from Equation (71), what are the final velocities and kinetic energies of each glider?

2. If two gliders of equal mass m and equal and opposite initial velocity v collide and stick together, using Equation (78), derive a formula for the total initial kinetic energy, the total final kinetic energy, and the amount of energy converted to heat, in terms of m and v.

ROTATING REFERENCE FRAMES

BACKGROUND

All measurements of the motions of objects must be made relative to a specified reference frame. The choice of reference frame is extremely important in formulating the laws of physics. This is because the laws of physics can often be formulated in a simple way for so-called *inertial* frames of reference (defined below); but for non-inertial frames the laws may be much more complicated.

Inertial reference frames. We begin with Newton's first law, which, in effect, defines a so-called inertial frame of reference:

> "A frame of reference is termed *inertial* if all objects, with respect to the reference frame and with all forces removed, remain either at rest or move in a straight line with constant velocity. Put another way, a frame of reference is termed inertial if all objects, with all forces removed, continually have zero acceleration with respect to the reference frame."

In other words, temporarily remove any physical forces acting on a body (such as gravity, electricity, magnetism, friction, air, string tension, *etc*.) from an object. If its acceleration is zero and remains zero, then the frame of reference is termed *inertial*.

The concept of an inertial reference frame is an idealization. For example, rotating reference frames are certainly not inertial frames; and since the earth is rotating about its axis, a frame of reference attached to the earth is not precisely inertial. Moreover, the earth is revolving about the sun, further compromising its inertial properties. Nevertheless, the accelerations associated with the rotation of the earth about its axis and the revolution of the earth about the sun are very tiny in comparison, say, to g, the acceleration of gravity,[21] so that we may say that for many experiments confined to the laboratory the earth is *approximately* an inertial frame of reference.

Newton's second law.

Newton's second law states that, with respect to an inertial frame of reference, if a force is applied to an object, the object will accelerate.[22] If the force is $\vec{\mathbf{F}}$ and the object has a mass m, then the acceleration is

$$\vec{\mathbf{a}} = \frac{1}{m}\vec{\mathbf{F}} \quad \text{(Newton's second law)} \tag{81}$$

This fundamental formula has been the basis of much of the work that you have done thus far. The prescription is simple in principle: Remove all the forces from an object, and test to see if the object does not accelerate (*i.e.* remains stationary or moves with uniform velocity in a straight line). If the object does not accelerate, the reference frame is *inertial*.

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[21] The magnitude of the acceleration of a person standing at the equator is approximately $a = v^2/r = .034$ m/s^2. This is much smaller than the gravitational acceleration constant g.

[22] Acceleration, of course, may be negative or positive.

You may then apply forces to the object, and it will accelerate according to Newton's second law.

If the object *does* accelerate without any forces applied, you are unfortunately measuring with respect to a non-inertial frame of reference, and Newton's second law is inapplicable. You will have to use more complicated rules, which we will discuss below.

In this laboratory, we are going to investigate the challenging problem of non-inertial reference frames. Since, with respect to a non-inertial frame, an object accelerates (by the very definition of a non-inertial frame), we can turn Newton's second law onto its head by inventing an *apparent* force that makes it *appear* that Newton's second law is true in that frame. In other words, after we have removed all known real forces (gravity, electricity, magnetism, *etc.*), since the object still accelerates, we make up a *pseudo-force* $\vec{\mathbf{F}}_{\mathbf{p}}$ defined as

$$\vec{\mathbf{F}}_{\mathbf{p}} \equiv m\vec{\mathbf{a}}_{ni} \tag{82}$$

where $\vec{\mathbf{a}}_{ni}$ is the acceleration of the object arising from the non-inertial nature of the reference frame.

To make these ideas more specific, let's consider a concrete example. Your friend has just bought a new car, which can accelerate uniformly in the +z direction, $\hat{\mathbf{e}}_{\mathbf{z}}$, from 0 to 60 miles per hour in just 10 seconds. Knowing that the interior of the car is not an inertial reference frame, you wish to formulate a modified version of Newton's second law.

Now it is easy to show that 60 miles per hour is exactly 88 feet per second, or exactly 26.8224 meters per second (please verify!).[23] Therefore, the car is accelerating at +2.68224 meters per second per second. Therefore, if you are riding in the passenger seat of the car and you left a coin in front of you on a frictionless dashboard, a careful measurement of the motion of the coin will show it flying off the dashboard and accelerating toward you with an acceleration of -2.68224 meters per second (the negative sign implies that the acceleration is toward the back of the car). Because the coin is accelerating in your reference frame (the car), there is an *apparent* force, called the *pseudo-force*, apparently acting on it, given by

$$\vec{\mathbf{F}}_{\mathbf{p}} \equiv m\vec{\mathbf{a}}_{ni} = -2.68224m\hat{\mathbf{e}}_{\mathbf{z}} \tag{83}$$

Now, if we apply additional, real forces to the object (such as friction, magnets, gravity, *etc*.), we must include the pseudo-force in calculating the acceleration with respect to our non-inertial reference frame. In other words, we can "pretend" that the interior of the car is an inertial reference frame, but that all objects in the car are now subject to an additional mysterious "force" that we were unable to remove. Moreover, we will need to apply a real, restoring force to these objects to prevent them from moving with respect to the interior of

[23] This is because the inch is defined as exactly 25.4 mm.

the car. In the case of our coin on the dashboard, some friction will be needed to keep the coin from flying into our face.

Motion in a rotating frame.

The uniformly accelerating frame discussed above is probably the simplest non-inertial frame. A more complicated non-inertial frame is the uniformly *rotating* frame, such as a merry-go-round or the earth, spinning with a constant angular velocity. Here also it is possible to construct an acceleration law that enables us to correctly determine the motion of an object with respect to the rotating frame.

The result, which is derived in the textbook, is that there are two new *apparent* forces that must be added to any existing real force in order to describe correctly the dynamics of an object of mass m moving in the rotating frame. These pseudo-forces are called the Coriolis pseudo-force and the centrifugal pseudo-force. They look like this:

$$\vec{\mathbf{F}}_{\mathbf{cor}} = -2m(\vec{\omega} \times \vec{\mathbf{v}}_{\mathbf{R}}) \text{ (Coriolis pseudo-force)} \tag{84}$$

$$\mathbf{F}_{\mathbf{cen}} = -m[\vec{\omega} \times (\vec{\omega} \times \vec{\mathbf{r}}_{\mathbf{R}})] \text{ (centrifugal pseudo-force)} \tag{85}$$

In these expressions, $\vec{\mathbf{r}}_{\mathbf{R}}$ is the (vector) position of an object *with respect to the rotating system*, and $\vec{\mathbf{v}}_{\mathbf{R}}$ is the vector velocity of the object, also with respect to the rotating system. We can now modify Newton's second law, as expressed above, to read as follows:

$$\begin{aligned} &\vec{\mathbf{a}}_{\mathbf{R}} = \frac{1}{m}\vec{\mathbf{F}}_{\mathbf{tot}} \Rightarrow \vec{\mathbf{a}}_{\mathbf{R}} = \frac{1}{m}\{\vec{\mathbf{F}}_{\mathbf{ext}} - 2m(\vec{\omega} \times \vec{\mathbf{v}}_{\mathbf{R}}) - m[\vec{\omega} \times (\vec{\omega} \times \vec{\mathbf{r}}_{\mathbf{R}})]\} \\ &\Rightarrow \vec{\mathbf{a}}_{\mathbf{R}} = \frac{1}{m}\vec{\mathbf{F}}_{\mathbf{ext}} - 2(\vec{\omega} \times \vec{\mathbf{v}}_{\mathbf{R}}) - [\vec{\omega} \times (\vec{\omega} \times \vec{\mathbf{r}}_{\mathbf{R}})] \end{aligned} \tag{86}$$

In these expressions, $\vec{\omega}$ is the (constant) angular velocity of the rotating frame.[24] It is a vector, which points along the axis of the frame, and its direction is determined by the right hand rule.[25] Its magnitude, ω, is measured in radians per second, a number proportional to the number of revolutions per second. Since there are 2π radians in one revolution, ω equals $2\pi/T$ where T is the number of seconds per revolution. The acceleration $\vec{\mathbf{a}}_{\mathbf{R}}$ is also measured with respect to the rotating reference frame.

The meaning of this equation is as follows: in a rotating reference frame, the acceleration of an object is given by the usual response to an external force, $\vec{\mathbf{F}}_{ext}$; in addition, however, there will be an acceleration *that does not depend upon the mass of the object*. This happens, of course, because the acceleration is furnished by the reference frame itself!

[24] There is a third apparent force that arises if the angular velocity ω is not constant. We will only consider *constant* angular velocity in this laboratory.

[25] Curl the fingers of your right hand in the direction of rotation. Your thumb will then point along the direction of $\vec{\omega}$.

We have succeeded in our program to construct the law of acceleration with respect to a uniformly rotating frame of reference. There are two important consequences to our work thus far. They are summarized as follows:

1. Newton's laws are, for our purposes, exact laws of physics. However, we are only entitled to use them on measurements made with respect to *inertial* reference frames. It turns out that we can always construct an infinite number of inertial reference frames,[26] so there never is a need to discard the laws.

2. If we foolishly insist on making measurements of position, velocity, and acceleration with respect to a non-inertial (accelerating or rotating) reference frame, Newton's laws do not apply. This is not because the laws are untrue; it is because we are insisting on using an invalid reference frame. Our laws of physics in an accelerating or rotating reference frame will be much more complicated than Newton's laws, because we will need to invent fictitious forces to explain the bizarre motions relative to our non-inertial frame.

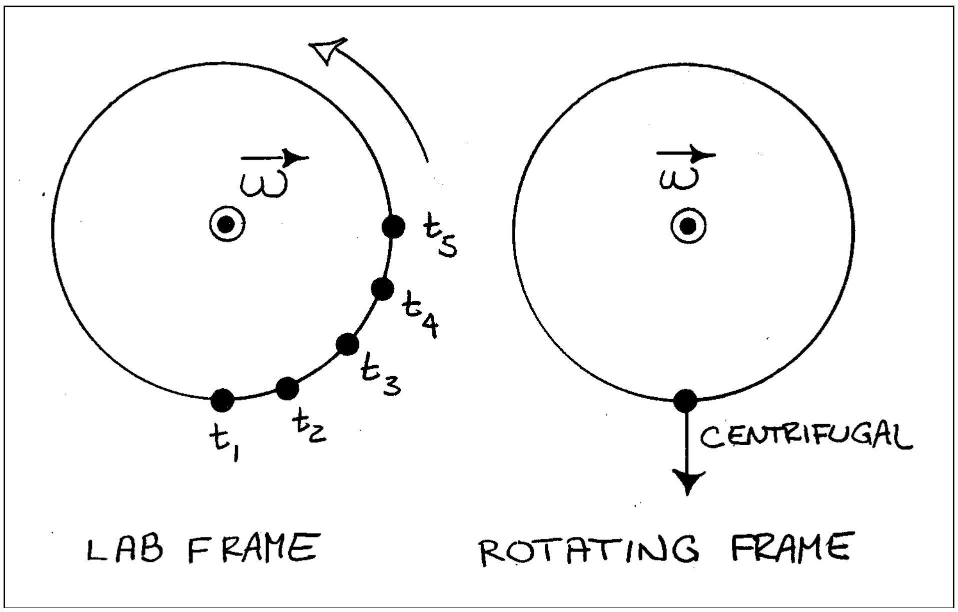

Figure 9. An object appears to experience the centrifugal pseudo-force but not the Coriolis pseudo-force when it is at rest in a rotating frame, such as a squirrel sitting on the rim of a merry-go-round. The centrifugal pseudo-force points radially outward and has magnitude $m\omega^2 r$. To keep the squirrel fixed in place, a real, frictional force, the centripetal force, must be applied to compensate for the centrifugal pseudo-force.

The centrifugal pseudo-force

[26] Any reference frame moving at a uniform velocity with respect to an inertial reference frame is also an inertial reference frame.

The centrifugal pseudo-force, $-m(\vec{\omega}\times(\vec{\omega}\times\vec{\mathbf{r}}_{\mathbf{R}}))$, is the simpler force to consider, because, unlike the Coriolis pseudo-force, it does not depend upon the object's velocity in the rotating frame. Moreover, it always points radially outward, and has a magnitude of $m\omega^2 r$, where r is the perpendicular distance of the mass from the axis of rotation. Since the speed v of a rotating mass is given by $v=\omega r$, the magnitude of the centrifugal force will be mv^2/r, which of course is the familiar force required to keep an object in circular motion. Also note that an object lying on, or flying over, the axis of rotation will experience no centrifugal pseudo-force at the moment or moments when $r = 0$.

The centrifugal pseudo-force appears to an observer on a rotating frame to act on an object whether or not there are other true forces acting on the object. Thus, a bird gliding in a straight line at uniform velocity across a merry-go-round appears to an observer on a merry-go-round to have the centrifugal pseudo-force acting on it.[27] A squirrel, sitting at rest on the rim of the merry-go-round (as illustrated in Figure 9, above) will experience the centrifugal pseudo-force since the squirrel is not at the center of the frame. However, since the squirrel is stationary, there must be a *true* force that balances the pseudo force; this is the real frictional force of the platform of the merry-go-round pushing the squirrel toward the center. This true force is given a name: it is called the *centripetal* force. If you are driving a car and you make a very sharp right turn, you will sense a centrifugal pseudo-force that appears to accelerate you toward the left door; the door in turn will push on you with a true force, the centripetal force, to keep you, hopefully, inside the car.

The Coriolis pseudo-force

The Coriolis pseudo-force, $-2m(\vec{\omega}\times\vec{\mathbf{v}}_{\mathbf{R}})$, is a pseudo-force that is proportional to the velocity of the object as measured in the rotating frame. Its direction is sideways; *i.e.* perpendicular to the velocity of the object. So if the object is not moving with respect to the rotating frame, the Coriolis pseudo-force is zero. A person who is stationary on a merry-go-round will not sense the Coriolis pseudo-force.

If the motion of the object is perpendicular to the axis of rotation, then $|\vec{\omega}\times\mathbf{v}_R| = \omega v$, and the magnitude of the Coriolis pseudo-force will be $2m\omega v = 2mv^2/r$. (Note that the magnitude of the Coriolis pseudo-force is *twice* the magnitude of the centrifugal pseudo-force). The direction will be perpendicular to the axis of rotation and perpendicular to the velocity vector.

Unfortunately, the only situation where the Coriolis pseudo-force acts alone (that is, without being accompanied by the centrifugal pseudo-force) is at the moment when the object passes over the axis of rotation. In all other cases, we will have to carefully keep track of both pseudo-forces.

A combination of pseudo-forces

We now present two examples of combinations of the Coriolis pseudo-force and the centrifugal pseudo-force, which we will demonstrate in the subsequent experiments.

---

[27] The bird will also appear to have the Coriolis pseudo-force acting on it, as will be discussed below.

Consider an object which is perfectly stationary in the laboratory frame; say a tree, or a puck floating motionlessly on a rotating table, as illustrated below in Figure 10. A moment's reflection will convince you that, as observed from a rotating coordinate system, the object will appear to move in a circle, rotating in the opposite sense as the frame of reference. Since the object is rotating with respect to the rotating coordinate system, it has a velocity vector which is tangential and of magnitude $|\mathbf{v}| = \omega r$. Therefore, the Coriolis pseudo-force points radially inward, and has magnitude $2m\omega^2 r$. In addition to the Coriolis pseudo-force, the object will appear to experience the centrifugal pseudo-force, because the object is not sitting on the axis. One can see that the centrifugal pseudo-force will point radially outward, with magnitude $m\omega^2 r$. The net pseudo-force acting on the object is therefore inward, and of magnitude $m\omega^2 r$. This is precisely the inward force required to keep an object of mass m moving in a circle with angular velocity ω.

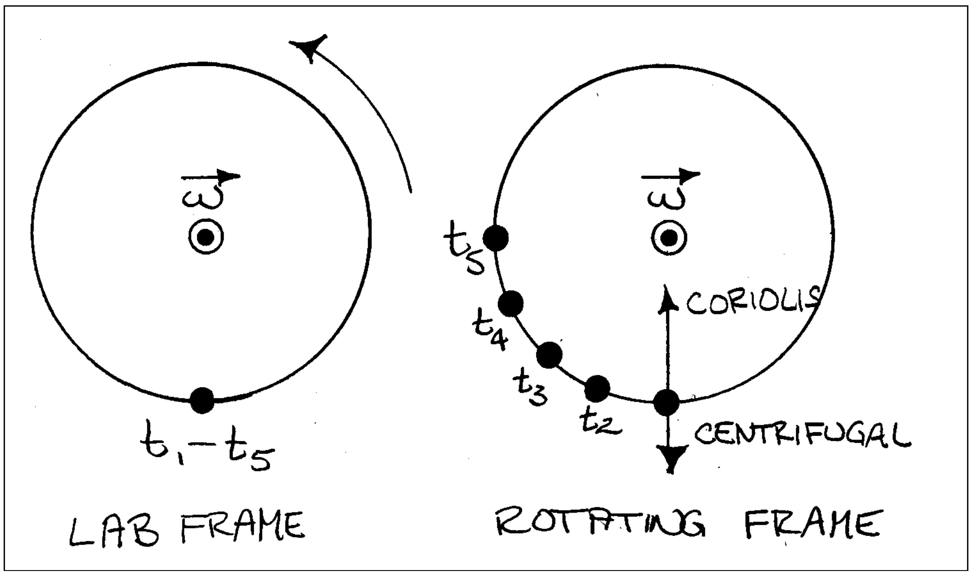

Figure 10. An object (like a tree or a puck floating on a frictionless table) which is stationary in the laboratory frame will appear to move in a circle in a rotating frame. Because it is moving in the rotating frame, it will appear to experience a Coriolis pseudo-force, of magnitude $2m\omega^2 r$, directed inward. Since it is at a fixed radius, it will also appear to experience a constant centrifugal pseudo-force, directed outward, of magnitude $m\omega^2 r$.

This example shows how we can take the simplest conceivable situation (a stationary object in an inertial reference frame) and make it appear really complicated! The fact that it works out so perfectly reinforces our confidence in the method of pseudo-forces, yet it encourages us, whenever possible, to work in inertial reference frames.

As a second example of a combination of the centrifugal and Coriolis pseudo-forces, consider a bird flying at uniform speed in a straight line over our merry-go round, as in Figure 11 below. To an observer on the merry-go-round, the bird will appear to move in a curved trajectory, leading the observer to conclude that sideways forces are acting on the bird. Since the bird is moving, we know that one of these pseudo-forces is the Coriolis pseudo-force, $-2m(\vec{\omega}\times\vec{\mathbf{v}}_{\mathbf{R}})$. Since $\vec{\omega}$ points upward along the axis of rotation, the Coriolis pseudo-force points laterally, perpendicular to the velocity vector.

In addition to the Coriolis pseudo-force, to an observer in the rotating frame, the bird appears to experience the centrifugal pseudo-force, $-m(\vec{\omega}\times(\omega\times\vec{\mathbf{r}}_{\mathbf{R}}))$, except at the precise moment when it flies over the axis of rotation. The centrifugal pseudo-force points outward, with a magnitude of $m\omega^2 r$. The total pseudo-force will be the vector sum of these two pseudo-forces, and will vary along the path of the bird.

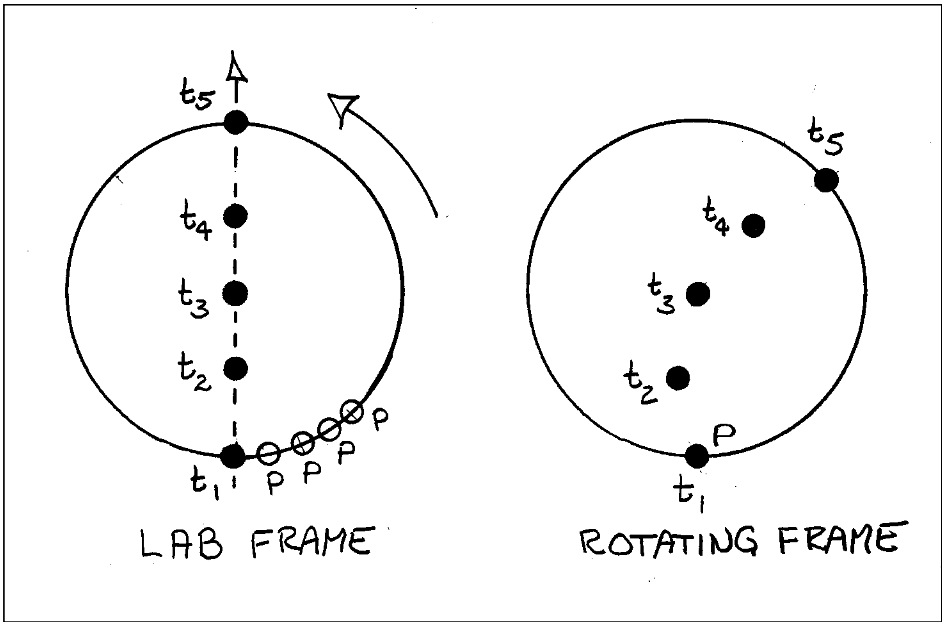

Figure 11. The trajectory of a bird flying at uniform speed in a straight line over a merry-go-round rotating in the counter-clockwise direction. The left panel shows the trajectory in the laboratory frame, and the right panel shows the trajectory as seen by an observer in the rotating frame. The open circles, labeled P, show the motion of a fixed point on the rim of the rotating reference frame. The point P moves counterclockwise in the lab frame, but of course it is stationary in the rotating frame. Because the bird travels in a curved trajectory with respect to the rotating frame, the rotating observer concludes that the bird is subject to lateral forces. In this case, the apparent lateral forces are a combination of the Coriolis pseudo-force and the centrifugal pseudo-force.

THE CENTRIFUGAL PSEUDO-FORCE

The centrifugal pseudo-force, as discussed in the introduction, appears, to observers on a rotating frame, to push an object radially outward, with an apparent force proportional to the distance of the object from the center of rotation.

Experiment 1.1 – *Object on the rim of a rotating table or merry-go-round*

To observe the centrifugal pseudo-force by itself, we need a situation where an object is motionless with respect to a rotating frame of reference. The simplest and most graphic example is when the observer rides on the rim of a merry-go-round or rotating table.

In the lab room:

1) Place the "observer" (a small doll or animal figure) on the rim of the table and spin the table. Does the observer feel an apparent force acting on them? In what direction is that force?
2) If you spin the table fast enough, what happens? Explain this in terms of the centrifugal pseudo-force and the true centripetal force.

At the merry-go-round:

1) Sit on the rim of the merry-go-round and have someone start it spinning.
2) Do you feel an apparent force acting on you, and, if so, what direction is that force? What is the direction of the true force acting back upon you, to keep you motionless the merry-go-round?

Experiment 1.2 – *Static Foucault pendulum*

The centrifugal pseudo-force may be also observed with the static *Foucault pendulum*. The Foucault pendulum is a simple pendulum that can be swung freely (without horizontal constraint) in any direction.

In the lab room:

1) With the pendulum centered **away** from the rotational axis and with the bob *stationary* with respect to the rotating table (not oscillating back and forth), slowly spin the table in the counterclockwise direction.
2) With the pendulum **not oscillating** back and forth, does it still hang vertically?
3) Adjust the pendulum support so that the pendulum is centered either closer or further from the rotational axis. How does its equilibrium position change with the radial position of the pendulum support?
4) From the perspective of **someone on the platform** (you may place the miniature observer on the platform to help visualize), does it seem that there is a force pushing the bob out an angle? From that same viewpoint, what is the net force on the bob?
5) From the viewpoint of an **observer in an inertial frame**, what is the net force on the bob?
6) Draw a free-body diagram of the hanging pendulum in both the rotating frame and an inertial frame, labeling all the appropriate forces. Solve the problem in both frames to find an expression for the deflection angle in terms of the angular velocity of the platform.

THE CORIOLIS (PLUS CENTRIFUGAL) PSEUDO-FORCE

As discussed in the introduction, there are no nontrivial examples of the Coriolis pseudo-force by itself. The Coriolis pseudo-force is always accompanied by the centrifugal pseudo-force,[28] so we must consider the two forces in tandem.

Experiment 2.1 – *Motionless air puck*

The simplest example of these two forces acting in tandem is the case of an object that is stationary in the laboratory frame.

In the lab room:

1) To demonstrate, place a motionless air puck on the counter-clockwise rotating table. In an ideal situation, the puck would remain stationary in the laboratory frame.[29]
2) Supposing that it does, sketch the trajectory that the puck follows in the table's frame. Denote on your sketch the direction and relative magnitudes of the centrifugal and Coriolis pseudo-forces.

Experiment 2.2 – *Object traveling across the rotating table/merry-go-round*

The second example of these two forces acting in tandem is the case of the puck gliding across the table.

In the lab room:

1) Try launching the puck diagonally across the table toward a partner opposite you. Does the puck travel in a straight line *with respect to the earth*, or does it curve sideways?
2) Does the puck travel in a straight line *with respect to the rotating table*, or does it curve sideways?
3) Sketch the trajectory that the puck follows in both the lab frame and in the rotating frame. Denote on your sketch in the rotating frame the direction and relative magnitudes of the centrifugal and Coriolis pseudo-forces.

At the merry-go-round:

1) You can recreate the above scenario in a different manner by tossing a beanbag diagonally across the merry-go-round toward a partner opposite you.
2) Does the bag travel in a plane *with respect to the earth*, or does it curve sideways?
3) Does the bag travel in a plane *with respect to the merry-go-round*, or does it curve sideways?
4) Does it matter if the bean-bag is pitched from the ground, or from the merry-go round?

Experiment 2.3 – *The dynamic Foucault pendulum*

The third example of these two forces acting in tandem is the case of the dynamic Foucault pendulum. The dynamic Foucault pendulum is a simple pendulum that can be swung freely

[28] Except at the instant when an object passes though the axis of rotation.

[29] Because the table is not perfectly flat and level, the puck may drift slightly in a random direction.

(without horizontal constraint) in any direction, specifically designed to demonstrate the rotation of the earth[30].

In the lab room:

1) To illustrate the behavior of a Foucault pendulum, set it up on the turntable when the turntable is not rotating. Does the pendulum swing back and forth in a plane?
2) Now slowly rotate the platform, so that the pendulum swings back and forth several times in one revolution of the platform.
3) From the standpoint of the laboratory, does the pendulum continue to swing in a plane, or does it have a more complicated motion?
4) From the standpoint of the rotating frame, does the pendulum continue to swing in a plane, or does the plane appear to rotate?

PRELAB QUESTIONS

1. Looking down from a stationary tree branch, a merry-go-round spins in a counter-clockwise direction with an angular velocity of 1 radian per second. A squirrel of mass 0.7 kg sits on the outer rim of the merry-go-round, at a radius of 4.2 meters.

a) What is the magnitude and direction of the vector $\vec{\omega}$?

b) What is the magnitude and direction of the Coriolis pseudo-force as felt by the squirrel?

c) What is the magnitude and direction of the centrifugal pseudo-force as felt by the squirrel?

2. Again, looking down from a stationary tree branch, a merry-go-round with a 1.4 meter radius spins in a counter-clockwise direction with an angular velocity of 1 radian per second. From your viewpoint, a bird of mass 0.5 kg flies in a straight line over the axis of the merry-go-round at a uniform speed of 3.2 m/s.

a) Draw the trajectory of the bird as seen from your stationary tree branch.
b) Draw the trajectory of the bird as seen from an observer on the merry-go-round.
c) Consider three instants:
   i. When the bird first crosses the outer edge of the merry-go-round;
   ii. When the bird crosses the center of the merry-go-round;
   iii. When the bird finally crosses the outer edge of the merry-go-round.

   For each of the three moments, as seen by an observer on the merry-go-round, illustrate the direction of the *centrifugal* pseudo-force that seems to act on the bird. At what point is the centrifugal pseudo-force 0? You may use your sketch from part (b).
d) For instant (i), also illustrate on your sketch the direction of the *Coriolis* pseudo-force acting on the bird (as seen with respect to the observer's merry-go-round reference frame).

[30] There is a large one of these in a glass cabinet in the back of Thimann 133.

ROTATIONAL DYNAMICS

Note: Offered only for Physics 5L

INTRODUCTION

In the previous laboratory we studied the kinematics arising from rotational motion. In other words, we studied the velocities and accelerations of objects arising only from changes in coordinate systems, from inertial to non-inertial and back. Dynamics is the study of how true forces (as distinct from pseudo-forces) affect the motion of objects.

If the force on an object is the amount of 'push' or 'pull' that it receives, then the torque on an object is the amount of twist that it receives. Since twisting requires a force to be exerted some distance off the axis of rotation, torque is manifested by a lever arm, as well as a force. Mathematically, we represent the torque as a cross-product:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \tag{87}$$

where $\vec{\mathbf{r}}$ is the displacement vector from the axis of rotation to the point at which $\vec{\mathbf{F}}$ is applied. From this, we concluded that the torque has units of force times length. Note that if the force is applied at the axis of rotation ($\vec{\mathbf{r}} = 0$), or if the force is directed in the same direction as $\vec{\mathbf{r}}$, then the torque is zero.

In translational dynamics, you learned from Newton that the acceleration of an object was directly proportional to, and in the same direction as the net force on an object, in accord with Newton's second law:

$$\mathbf{a} = \frac{1}{m}\mathbf{F} \tag{88}$$

There is a similar principle in rotational dynamics that relates the angular acceleration $\vec{\alpha}$ to the torque $\vec{\tau}$, that is often called Newton's second law for rotations:

$$\boldsymbol{\alpha} = \frac{1}{I}\boldsymbol{\tau} \tag{89}$$

The angular acceleration, $\boldsymbol{\alpha} \equiv d\boldsymbol{\omega}/dt$, is defined as the rate of change of $\boldsymbol{\omega}$, the angular velocity. The constant of proportionality, I, is the rotational analog of mass. If mass is a measure of an object's resistance to *translational* acceleration, then its moment of inertia is its resistance to *angular* acceleration.

The moment of inertia of a point mass, rotating about a fixed point a distance R away, is given by $I = MR^2$. To determine the moment of inertia of an extended body, such as a disk, one must add up the moments of inertia of all the constituent particles, usually by performing the mathematical operation of integration. The result of this process for a solid cylinder or disc of radius R, total mass M, and uniform mass distribution is given by

$$\boxed{I = \frac{1}{2}MR^2 \quad \text{(cylinder or disc of uniform density)}} \tag{90}$$

Having defined torque as the rotational analog of force, we can also define the rotational analog of momentum. Angular momentum, $\mathbf{L}$, is defined in terms of momentum in the same way as torque is defined in terms of force:

$$\mathbf{L} \equiv \mathbf{r} \times \mathbf{p} \tag{91}$$

Again, $\mathbf{r}$ is the displacement vector from the axis of rotation to the particle of momentum $\mathbf{p}$.

You may remember an alternative statement of Newton's second law in terms of the time rate of change of the momentum:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \tag{92}$$

Newton's second law for rotations can be expressed in a similar form:

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} \tag{93}$$

Note that if there is no torque, then $d\mathbf{L}/dt = 0$, so $\mathbf{L}$ = (constant). This is the principle of the conservation of angular momentum, which you will verify later in this lab. Combining the two previous equations, we get

$$\frac{d\mathbf{L}}{dt} = I\boldsymbol{\alpha} \tag{94}$$

Or, by substituting $\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$ and integrating,[31] we get

$$\mathbf{L} = I\boldsymbol{\omega} \tag{95}$$

This is a second, equally valid, expression for the angular momentum of an object, provided that th e axis of rotation passes through the center of mass of the object.

Finally, we cannot ignore the fact that we have been dealing with vector quantities, and so we must pay careful attention to directions. Now, at any given moment, different parts of a rotating object are moving in different directions. However, the axis about which all these

[31] We are assuming that the object is rigid, so I is a constant in time. Also, we are defining the angular momentum to be zero when the object is not rotating.

parts are rotating is often stationary. Thus, the direction of $\vec{\omega}$ is along the axis of rotation, with sign determined by the right hand rule.[32]

We define the direction of torque and angular momentum in much the same way. From the preceding discussion, angular momentum always points in the same direction as angular velocity. Torques that increase the angular momentum point in the same direction as $\vec{L}$, while torques that decrease the angular momentum are pointed in the opposite direction.

Apparatus

The apparatus that we will use consist of two metal discs, spinning about a fixed vertical axis, as shown in Figure 12. The discs spin on air bearings to reduce friction. In this sense, they are the rotational analog of the one-dimensional air track.

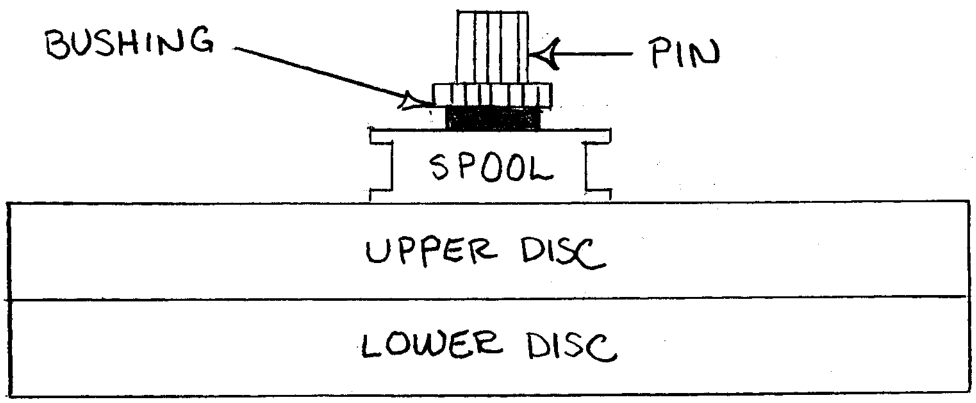

Figure 12. Components of the angular momentum apparatus.

In these experiments, the lower disc will always be a steel disc, and it is the only disc designed to serve as the bottom disc. You will never have any need to remove the bottom disc. For the upper disc you will have two choices: steel or aluminum. Always store the unused discs on the felt pads at your bench.

Air pressure is supplied to the system through a pressure regulator. The main compressor provides between 80 and 100 psi of air pressure to the regulator. The regulator should be adjusted to between 3 and 10 psi by turning the yellow cap clockwise for higher pressure, and *vice versa*. If the yellow cap does not turn, it is probably locked. To unlock it, lift it upward.

There are two air bearings in this apparatus; the lower air bearing supports the lower steel disc, and the upper air bearing supports the upper disc on top of the lower disc. For our experiments, the lower air bearing will always be on. However, the upper air bearing is activated by placing the pin into the hole at the top of the axis.

[32] Curl the fingers of your right hand in the direction of rotation. Your thumb will then point along the direction of $\vec{\omega}$.

To apply a torque to the upper disc, we must supply a force, acting on some point away from the axis of rotation. This force will be supplied by suspending a 25.0 gram mass on a string, looping it over an air bearing, and connecting it to the spool that can be fastened to the top disc. Once the string is connected, you should be able to wind it on to the spool. When you let go, the mass should fall downward while spinning the disc.

Along the edge of the spinning discs, you will find a pattern of exactly 200 vertical black bars. Precisely every two seconds, the electronic digital counter counts the number of bars, divides by 2.000, and displays the result. Therefore, the counter displays the number of bars per second, and refreshes the display every *two* seconds.

To obtain the angular velocity of the top or the bottom register, you need to recall that one revolution of a disc is 2π radians and that it corresponds to 200 counts (see above). Therefore,

$$\omega = (\text{register reading in counts/sec}) \times \left(\frac{2\pi \text{ radians}}{200 \text{ counts}} \right) \tag{96}$$

CONSERVATION OF ANGULAR MOMENTUM

With a good feeling for torque and moment of inertia, we can now discuss rotational dynamics. The first case we will consider is when $\vec{\tau} = 0$, so that $\vec{\mathbf{L}} = I\vec{\omega}$ is constant in time In this experiment, we are going to use the rotational dynamics apparatus to verify that, in the absence of any external torques, angular momentum is conserved.

> **PLEASE NOTE:**
> **The small aluminum SPOOL and black bushing must be installed for all experiments; the instrument won't work properly without them**! **Please refer to** Figure **12**.

Experiment 1 – *Steel top disc*

Setup: Install the top steel disc on top of the (permanent) bottom steel disc. Then set the small aluminum take-up spool on top of the top steel disc and fasten it with the black bushing.[33] Then, insert the pin that is normally in storage into the bushing. This will allow the top disc to spin freely above the lower disc. You will need to adjust the fine pressure regulator to somewhere between 3 and 10 psi. to accomplish this.

Procedure:

1) While gently holding the bottom disc stationary, start the top disc spinning, and measure its initial velocity.
2) While the top disc is still spinning, turn off the air bearing between the two discs by pulling the pin from the bushing, so that the discs stick together and rotate as one. Now measure the final angular velocity of the combined disc system.

[33] For this experiment, you should remove the string assembly and set it aside.

Since the bottom disc was initially at rest, its initial angular momentum was zero. Therefore, all the initial angular momentum was in the top disc.

3) Calculate this initial L, using the known mass of the top steel disc (M = 1.479 kg) and the known radius (R = 63.3 mm.)
4) Compare your initial L to the final angular momentum, after the two discs have stuck together. For this you will need to know the mass of the lower steel disc (M = 1.468 kg). Are the two angular momenta equal?

Experiment 2 – *Aluminum top disc*

Setup: Install the top Aluminum disc (M = 0.461 kg.) on top of the (permanent) bottom steel disc. Then set the small aluminum take-up spool on top of the Aluminum disc and fasten it with the black bushing. Then, insert the pin that is normally in storage into the bushing. You may have to lower the pressure for the top Aluminum disc to stop spinning freely when the pin is released.

Procedure: Repeat the procedure from **Experiment 1**. Again, is angular momentum conserved?

TORQUE AND ANGULAR ACCELERATION

Now we will investigate a system where the net external torque is not zero. For this experiment, you will need to use the *steel* top disc. The basic setup is shown in Figure 13.

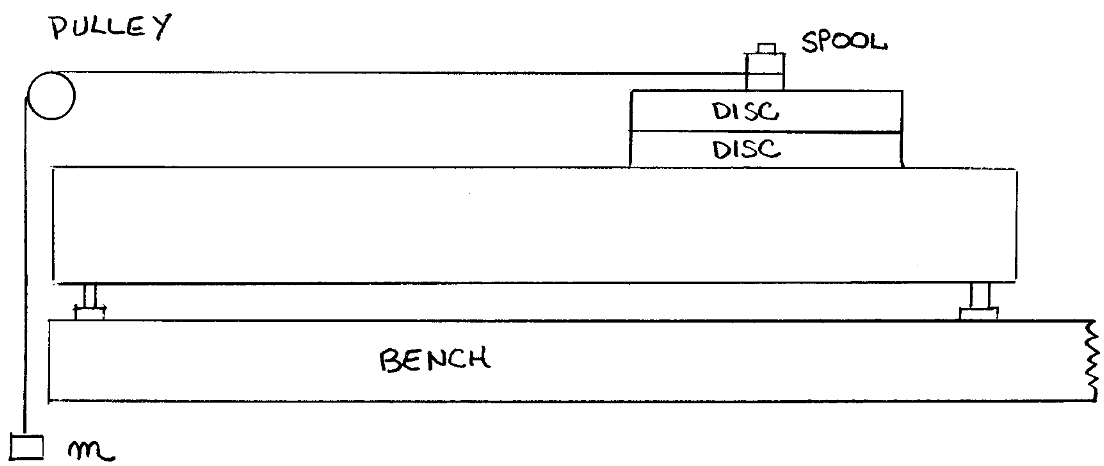

Figure 13. The angular acceleration apparatus

Experiment 3 – *Torque and angular acceleration from a hanging mass*

Setup: Remove the center pin so that the two steel discs spin in unison. Attach a string to the spool; loop the string over the air bearing pulley, and attach a 25 g mass to the lower end of the string. By gently turning the discs with the air bearing on, wind the thread around the spool until the top of the 25 g mass is roughly level with the bench top, hanging just below the cylinder air bearing.

Procedure:

1) With your hand, hold the discs stationary for a moment and then release them. The falling mass will accelerate the discs. When all the thread has unwound from the pulley, the mass will reverse direction and the thread will wind up again on the pulley.
2) As the string unwinds, record the counter value at each two second interval, and calculate ω for each counter reading. This will give you the angular velocity of all the discs as a function of time.
3) Read through the **Analysis** section below.
4) From your data, plot $d\theta/dt$ (recalling that $d\theta/dt = \omega$) as a function of time,[34] draw a best-fit line[35], and compare the slope of the line that you obtain with the predicted value. Are they in reasonable agreement? Note that since we know m, r, and I very well, we could just as well use this as a way to measure the acceleration of gravity at the earth's surface.

Analysis: The analysis of this system is very similar to the Atwood machine in the second laboratory, in that we have a falling mass tugging on an object (this time rotating) with inertia. Let us define the tension in the string to be T, where we allow for the possibility that the tension in the string can change with time. Also, let us use the symbol m for the 25 gram mass, and z for its vertical position relative to its starting point (our convention will be that downward motion of the mass is positive). Then, Newton's second law, applied to the free-body diagram *for the falling mass,* gives

$$ma = F \quad \Rightarrow \quad m\frac{d^2z}{dt^2} = mg - T \tag{97}$$

Now shift your attention to the *discs*. Suppose that they have a combined moment of inertia I, and that the spool has a radius r where the string is wound (Use $r = 1.25$ cm). The moment of inertia for a disc of mass M and radius R is given by

$$I = \frac{1}{2}MR^2 \tag{98}$$

Since the torque on the discs will be $\tau = Tr$, from Newton's second law for rotational motion, we have[36]

$$I\alpha = \tau \quad \Rightarrow \quad I\frac{d^2\theta}{dt^2} = Tr \quad \Rightarrow \quad \frac{I}{r}\frac{d^2\theta}{dt^2} = T \tag{99}$$

The way we connect this formula to the previous one is to note that the linear motion of the mass is connected to the rotary motion of the spool by $z = r\theta$. Therefore, formula (97) reads

---

[34] Be sure to note that the register updates every *two* seconds!

[35] This best-fit line does not have to go through the origin.

[36] We have dropped the vector signs in this expression, because all of the vectors are co-linear and point in the same direction (vertical).

$$m\frac{d^2(\theta r)}{dt^2} = mg - T \quad \Rightarrow \quad mr\frac{d^2\theta}{dt^2} = mg - T \tag{100}$$

Therefore, adding equations (99) and (100), *T* cancels out, and we get

$$\left(mr + \frac{I}{r}\right)\frac{d^2\theta}{dt^2} = mg \quad \Rightarrow \quad \frac{d^2\theta}{dt^2} = \frac{mgr}{mr^2 + I} \tag{101}$$

Now, since the mass of the weight, 25 grams, is much smaller than the mass of the discs, and because the radius of the spool, r, is much smaller than the radius of the discs, we can neglect mr^2 in comparison to I in the denominator (refer to the prelab questions for an explicit verification of this).[37] Therefore, upon a single integration, we have the following result:

$$\frac{d^2\theta}{dt^2} = \frac{mgr}{I} \quad \Rightarrow \quad \frac{d\theta}{dt} = \frac{mgr}{I}t \tag{102}$$

This is the result that we want. Our register measures a quantity proportional to $d\theta/dt$, so if we convert our register reading to units of $d\theta/dt$ and plot versus time, **the result should be a line with slope mgr/I.**

[37] Since the spool has a mass on the order of 10 g, we can also ignore the contribution of the spool's moment of inertia to the total moment of inertia I.

PRELAB QUESTIONS

In the following problems, assume the following parameters:

Mass of the bottom steel disc: 1.468 kg.
Mass of the upper steel disc: 1.479 kg.
Mass of the upper aluminum disc: 0.461 kg.
Radius of each disc: 63.3 mm.
Radius of the take-up spool: r = 1.25 cm.
Mass of the falling weight: 25 grams.

1. Using the values given above, calculate the following:
a) The moment of inertia of the lower, steel disc.
b) The moment of inertia of the upper, steel disc.
c) The moment of inertia of the upper, aluminum disc.
d) The combined moment of inertia of the two steel discs.

2. Using the results of problem 1, for two steel discs, what percentage error is made by neglecting mr^2 with respect to I in equation (101)?

3. Suppose that the counter is reading 400 counts per second at some moment. What is the value of ω at that moment?

THE HARMONIC OSCILLATOR

NOTE: FOR THE 6L COURSE ONLY

INTRODUCTION

One of the delightful aspects of physics arises from the fact that the descriptions of extraordinarily complex natural phenomena can be provided by remarkably simple models. The harmonic oscillator is the most ubiquitous of such models. We will study two examples of harmonic motion: the mass on a spring, and the simple pendulum. A deep understanding of these systems will lead to a better understanding of the vibrational modes of atoms within molecules, and ultimately a better understanding of sound and light.

A MASS ON A SPRING

We will first study the motion of a mass attached to a spring, suspended in a gravitational field. Note that our convention for this lab is that for forces and displacements, the *downward* direction is the *positive* direction, so that the force of gravity on the mass, which is downward, is positive.[38] As shown in **Figure 14**, if a mass m is suspended in equilibrium by the spring, the mass will rest at l_1, *as measured from the suspension hook*. If the mass is removed, we define the position of the end of the spring to be l_0.

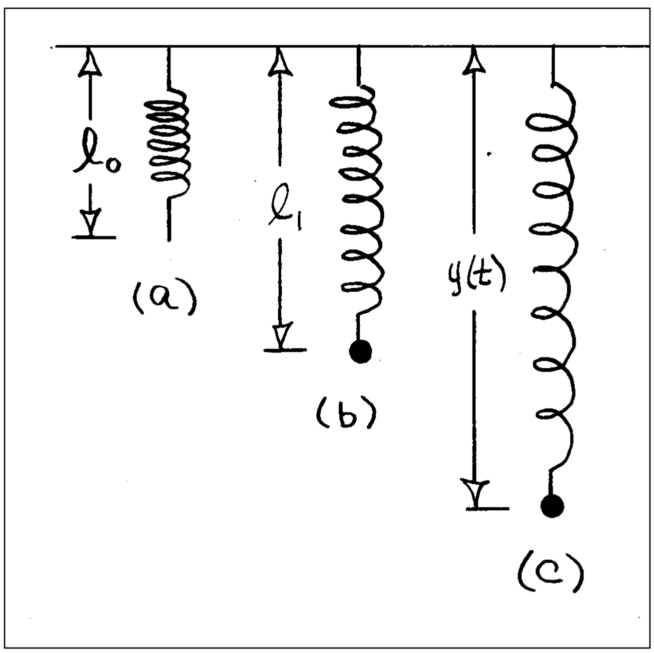

Figure 14. Parameters for the mass on a spring. Frame (a) is the spring without a mass; Frame (b) is the spring with mass attached but in stationary equilibrium; and Frame (c) is the mass in motion at an arbitrary position with respect to the suspension point.

If the spring constant is k, then the spring force on the mass at position l_1 will be $-k(l_1 - l_0)$ since the spring has been stretched by the positive distance $l_1 - l_0$. At the same time, the

[38] In class you may have used the opposite convention. As long as you are consistent throughout the analysis, the convention does not matter.

downward gravitational force on the spring will be just $+mg$. Thus, at equilibrium, these forces sum to zero:

$$\begin{aligned} &-k(l_1 - l_0) + mg = 0 \\ &\Rightarrow l_1 = l_0 + \frac{mg}{k} \end{aligned} \qquad (103)$$

We thus see that the effect of the force of gravity is to stretch the spring by an additional distance $+mg/k$.

At $t = 0$ we pull the mass down to a total distance $y(t = 0)$ *as measured from the support hook*, and release it from rest. At any time t after we have released the mass, there will be a net force acting on it, which, according to Newton's second law, will result in an acceleration given by

$$\begin{aligned} &m\vec{a} = \vec{F}_{gravity} + \vec{F}_{spring} \\ &m\frac{d^2 y(t)}{dt^2} = +mg - k[y(t) - l_0] \end{aligned} \qquad (104)$$

It is very convenient to measure the displacement of the mass relative to its equilibrium displacement l_1; we will call this relative displacement $u(t)$:

$$\begin{aligned} &u(t) = y(t) - l_1 = y(t) - (l_0 + mg/k), \text{ or} \\ &y(t) = u(t) + (l_0 + mg/k) \end{aligned} \qquad (105)$$

We can then substitute this expression for $y(t)$ in Equation (104), neatly eliminating the superfluous l_0 and l_1, and leading to the very simple result

$$\begin{aligned} &m\frac{d^2 u(t)}{dt^2} = +mg - k[(u(t) + mg/k + l_0) - l_0] = -ku(t) \\ &\Rightarrow \quad \frac{d^2 u(t)}{dt^2} + \frac{k}{m}u(t) = 0 \end{aligned} \qquad (106)$$

This is the famous harmonic motion equation. It states that the acceleration of a mass is proportional to, but in the opposite direction of, its position relative to its equilibrium position. To a good approximation, this equation applies equally well to quarks in the proton; to the relative motions of atoms in molecules, and to the motions of molecules within solids.

It is customary at this point to define a new symbol, ω_0, by the following:

$$\boxed{\omega_0 \equiv \sqrt{\frac{k}{m}}} \qquad (107)$$

Our equation of motion then looks even simpler:

$$\boxed{\frac{d^2u}{dt^2} + \omega_0^2 u = 0} \tag{108}$$

This is an equation that should be familiar to you from your elementary calculus course; it has the general solution (please verify!)

$$u(t) = A\cos(\omega_0 t + \phi) \tag{109}$$

and, by explicit differentiation the velocity is given by

$$\frac{du}{dt} = v(t) = -A\omega_0 \sin(\omega_0 t + \phi) \tag{110}$$

where A and ϕ are constants that depend upon the initial conditions. To determine the values of A and ϕ, we recall that in our case, at $t = 0$ we released the mass from rest at a position $y = y(0)$, corresponding to $u(0) = y(0) - l_1$. We also recall that we released the mass at $t = 0$ from *rest*. This latter condition forces us to set $\phi = 0$, or else we would have a non-zero velocity at $t = 0$ from Equation (110). Thus, our final result is

$$\begin{aligned} u(t) &= u(0)\cos(\omega_0 t) \\ \frac{du}{dt} &= -\omega_0 u(0)\sin(\omega_0 t) \end{aligned} \tag{111}$$

Thus, we see that the mass oscillates about its equilibrium position with an amplitude that is given by the amount of the initial manual stretch, $y(0)$- l_1; and that it oscillates with an angular frequency ω_0. The maximum speed of the mass is just

$$\boxed{\left|\frac{du}{dt}\right|_{\max} = \omega_0 \left|u(0)\right|} \tag{112}$$

From the above expression we see that the oscillation repeats itself in a time T, given by $\omega_0 T = 2\pi$; hence the period T is given by

$$\boxed{T = \frac{2\pi}{\omega_0}} \tag{113}$$

and the frequency $f = 1/T$ is given by

$$\boxed{f = \frac{1}{T} = \frac{\omega_0}{2\pi}} \tag{114}$$

Notice that the frequency (and hence the period and the angular frequency) depends only upon the spring constant k and the mass m. The frequency does not depend upon how much we stretched it by hand to get it started, nor does it depend upon the gravitational constant *g*. Gravity solely affects the *equilibrium position*, but not the frequency. We could transport our oscillator to the moon, or to intergalactic space for that matter, and it would oscillate at the same frequency.

Experiment 1 – Finding the spring constant k

Setup: You will find at your workbench a spring, a short rectangular rod, and the familiar photogate timer. A metal strip, or "fin", is attached to the rod, forming a cross which you can use to measure both the period of oscillation as well as the velocity of the mass at any point along its trajectory. The mass of the rod and flag assembly (including the connector to the spring) is 32.5 grams. A "stabilizer" consisting of a base with two narrowly separated rods is also included to prevent the rod and flag assembly from twisting as it moves up and down.

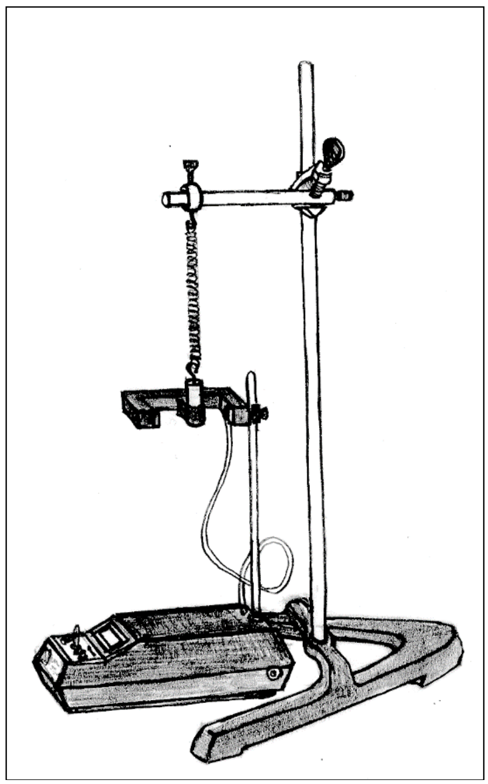

Figure 15. Mass on a spring, with photogate timer.

Procedure:

1) Read through the analysis below. Using the setup shown in **Figure 16**, calculate the spring constant *k*.

Analysis: The best way to calculate the spring constant is to measure the change in height Δy of the bob when you attach an additional $\Delta m = 10$ gram mass to it.[39] The spring constant k can then be determined by solving the spring equation $F_{spring} = -ky$, where F_{spring} is the restoring force of the spring on the mass, and is always opposite the displacement, y, of the spring (hence the minus sign).

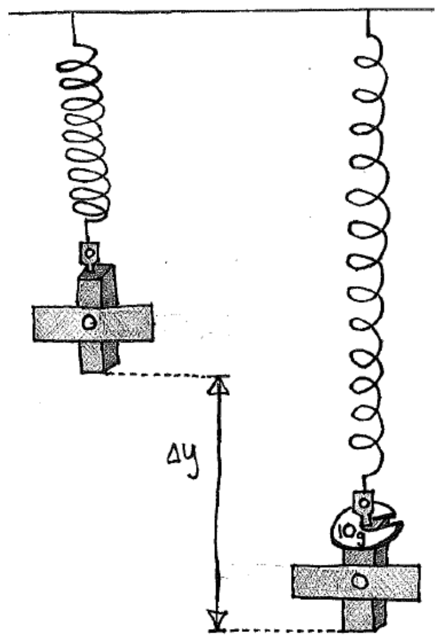

Figure 16. Adding the 10 gram mass to the spring assembly to obtain Δy

Also, recall that our convention is that the downward direction is the positive direction, so that the gravitational force acting on the mass is $F_{gravity} = +mg$. So the two forces acting on the bob sum to zero:

$$\begin{aligned} &F_{spring} + F_{gravity} = 0 \Rightarrow ky = mg \\ &\Rightarrow y = \frac{mg}{k} \end{aligned} \tag{115}$$

Now, we add a small mass (10 grams) to the initial mass:

$$\begin{aligned} &y + \Delta y = \frac{(m+\Delta m)g}{k} = \frac{mg}{k} + \frac{(\Delta m)g}{k} \\ &\Rightarrow \Delta y = \frac{(\Delta m)g}{k} \\ &\Rightarrow k = \frac{\Delta m}{\Delta y} g \end{aligned} \tag{116}$$

[39] You might ask why we don't just measure the total displacement of the bob when it is attached to the spring. The reason is that the force law for the springs that we use is not exact when the spring is fully compressed.

Experiment 2 – *Finding the period T*

1) From the spring constant and the mass of the bob, predict the period of the oscillator using equations (107) and (113).
2) To measure the period of the oscillator, use the photogate timer in **PEND** (pendulum) mode, and the **MODE** switch in **MEMORY** [40] mode. Make a series of measurements of the period as a function of the amplitude of the oscillation of the mass. Do your results agree with Equation (113)?
3) (*Optional*) For the previous derivations and calculations, we have assumed a massless spring. However, our spring is not massless—it has a mass m_s of approximately 12 grams (you may check the mass of your spring with the scale in the back of the lab room). The effective additional mass of the spring is just $m_s/3$. Thus, the characteristic angular frequency is predicted to be

$$\omega_0' = \sqrt{\frac{k}{m + m_s / 3}} \tag{117}$$

Using ω_0', now predict the period. Is this a closer match to your experimental value for T?

THE SIMPLE PLANE PENDULUM

The ideal simple pendulum comprises a point mass m suspended from a fixed point in space by a massless string or rod of length l. The pendulum is free to oscillate in a plane about its equilibrium position, with oscillation angle $\theta(t)$. The differential equation for the oscillation angle is derived in the textbook:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0 \tag{118}$$

This differential equation is exact, and it even applies to the situation where the pendulum swings rapidly in a circle! The general solution to this equation is very complicated, leading to the invention of the so-called elliptic integrals. However, for the classical "grandfather clock" pendulum, where θ never exceeds about 20 degrees, we may approximate, to very high accuracy (with θ expressed in radians!),[41]

$$\sin\theta \approx \theta \tag{119}$$

Our differential equation then becomes

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 \tag{120}$$

[40] See the Appendix on the photogate timer for details. The **MEMORY** function, among other things, *freezes* the first register even if the spring continues to oscillate and "trip" the photogate beam.

[41] The percentage error in this formula is about $\theta^2/6$, which at 20 degrees is about 2%.

This is same equation as the mass on a spring discussed earlier. In the words of Richard Feynman, “the same equations have the same solutions.” The characteristic angular frequency in this case is $\omega_0 = \sqrt{g/l}$. Thus the pendulum will oscillate about its equilibrium position, with a frequency (and thus period) that depends only upon the acceleration of gravity, *g*, and the length of the pendulum. In contrast to the mass on a spring, the pendulum’s frequency depends in a fundamental way on the action of gravity. After all, if there were no gravitational force, the pendulum would have no reason to oscillate. To the extent that the amplitude of the pendulum is small, the period is independent of the amplitude.[42]

Experiment 3 – The pendulum as a harmonic oscillator

Setup: At your bench you will find the ingredients for a pendulum, with two choices of *l* and two choices of *m* (you have no choice for the gravitational constant!). The mass of the aluminum bob is 24.1 grams, and the mass of the brass bob is 69.5 grams. The period of the pendulum can be measured by setting the photogate timer to **PEND** (pendulum) mode, and by setting the **MODE** switch to **MEMORY**.[43]

Procedure:

1. Design and perform a series of experiments that demonstrate the dependence of the period *T* upon:
   i. The length of the string
   ii. The mass of the bob
   iii. The amplitude of the oscillation.

   Discuss your results.

PRELAB QUESTIONS

1. Verify that the formula $u(t) = A\cos(\omega_0 t + \phi)$ is a solution to the differential equation for the mass on a spring, by plugging this expression for *u(t)* directly into the differential equation (108).

2. A mass suspended by a spring stretches an additional 6.2 cm when an additional 12.5 gram mass is attached to it. What is the value of the spring constant *k*, in SI units?

3. Calculate the angular frequency, the frequency, and the period of a simple pendulum consisting of a bob of mass of 0.085 kg suspended by a massless string of length 0.6 m.

---

[42] L. Euler showed that the percentage correction to the period, for small oscillations, is about $\theta_{max}^2/16$, which at 20 degrees of amplitude is about 0.8%.

[43] See the Appendix on photogate timers for details. The MEMORY function, among other things, *freezes* the first register (but not the second!) even if the pendulum keeps bobbing back and forth.

APPENDIX A: THE PASCO PHOTOGATE TIMER

Almost every laboratory section in this course makes use of a versatile instrument, called the *photogate timer*. This instrument, in its most basic form, can be used as a stopwatch, with 10^{-4} second resolution. However, its full power is realized when it is used to time processes that open or block a tiny infrared beam passing between the arms of the horse-shoe-shaped bracket.

The timer's control switches determine how it reacts to an object blocking its beam. The controls, moving from right to left, have the following actions:

RESET: This button resets all registers.

START/STOP: This button is one means of starting or stopping the timer. When the button is pressed for some time interval Δt, it is equivalent to blocking the light beam for the interval Δt (see below).

MEMORY: If one wishes to record two successive time intervals, set the **MEMORY** switch to **ON**. Ordinarily, the readout will display the first time interval after the unit has been reset. If the **MEMORY** switch is *on*, then a hidden register keeps track of the *sum* of the times of two successive intervals. This hidden register may be viewed by holding the **MEMORY** switch in the **READ** position.

Important note: We will *always* (unless specifically noted otherwise) use the timer in **MEMORY** mode, even though we may or may not be using the memory register. This way, the first register of the photogate timer will not be affected by subsequent interruptions of the light beam. However, the second, "hidden register," will unfortunately be dependent upon the sequence of subsequent interruptions. Therefore, if you plan to use the contents of the hidden register, be sure to not allow the flag to interrupt the beam more times than necessary to make your reading.

The leftmost control of the timer is a four-position mode switch. The four positions are **OFF**, **GATE**, **PULSE**, and **PEND**:

OFF: Power to the timer is switched off.

GATE: If the light is first blocked, then unblocked, the register displays the elapsed time between the blocking and unblocking of the light.[44]

PULSE: The timer is started the moment that the light is first blocked; the timer is then stopped at the moment when the light is blocked a second time. In other words, if we suppose that a pulse is created whenever the light is blocked, in pulse mode the register records the time between successive pulses.

[44] If one likens the blocking of the beam to the closing of a gate, then GATE mode measures the time interval over which the gate is closed.

PEND: This is the 'pendulum' mode. It is similar to the PULSE mode. The timer is started the moment that the light is first blocked; but the timer ignores the second blocking of the light, and then stops at the third blocking of the light. **Figure 17** displays the time interval displayed on the register for each of the modes described above.

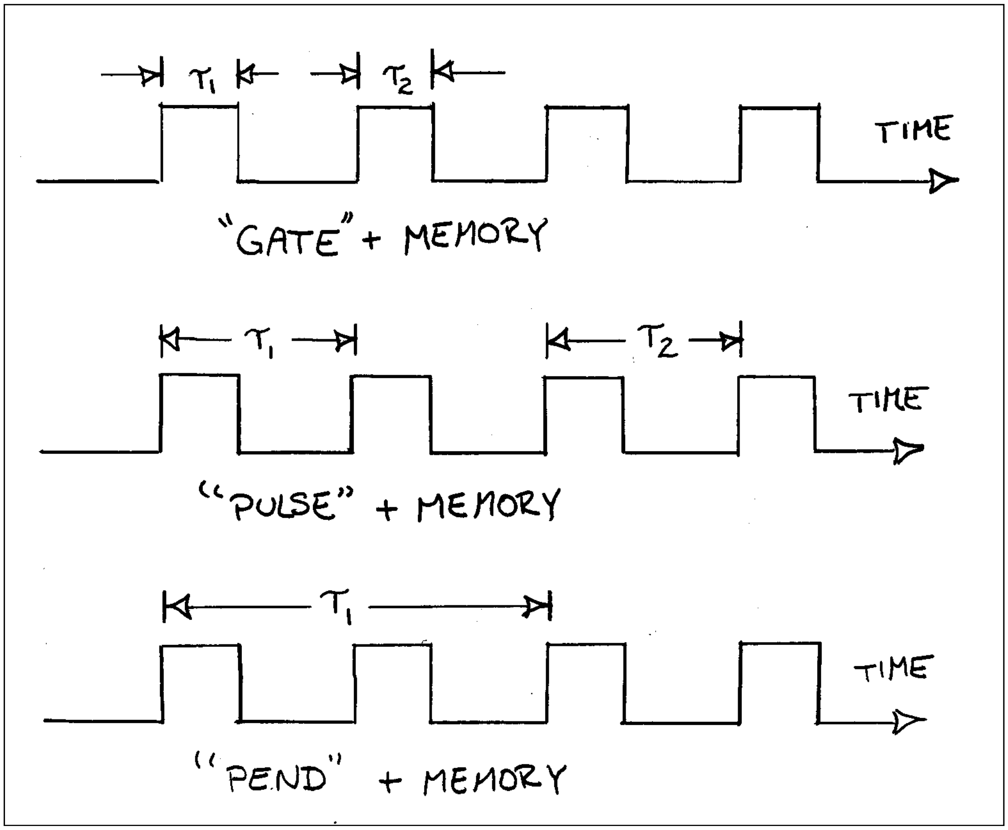

Figure 17. The timing diagram for the photogate timer.

APPENDIX B: GUIDELINES FOR LABORATORY NOTEBOOKS

As we mentioned in the introduction, you may be surprised to discover that the physics laboratory work is not conducted according to a cookbook: we do not offer recipes, and we do not expect that the final dish will be perfect! Your goal should not entirely be to find the “right answer;” rather, it should also be to understand the physical principles that led to the actual results that you recorded. Of course, this does not exempt you from meticulously planning and executing your experiments.

In this course there will not be time to produce a polished laboratory report. In the place of such a report, we would like you to submit an account of the work that you planned and performed, as if you were performing original research in a scientific laboratory. For full credit, your submissions should in some manner address the following topics:

1. Pre-laboratory report (submitted at the beginning of the section on separate notebook paper).
2. Account of your experiment: what you did.
3. Narrative of your predictions: what you expected to happen.
4. Account of your results: what happened.
5. Analysis: why you think it happened.

Since a typical lab consists of two or more subsections, the lab notebook need not contain this information in the order presented it above. In what follows, we describe in more detail our expectations.

1. Pre-Laboratory Report. The prelab questions are for the most very straightforward. In many cases they are ‘dry lab’ questions, and are intended to give you a ‘heads up’ on the kinds of calculations that you will need to do to complete a successful lab. If you wait until the last minute to do your prelab, you will not only hurt your prelab grade; you will harm your prospects of doing a masterful job on the lab itself.

2. Account of your experiment: what you did. Here there should be sketches of the most important parts of the apparatus, and there should be a narrative describing what was done with the apparatus. Again, this is not necessarily a separate section, so much as a requirement that the write-up continuously state what was actually done. Here is how the assessment is broken out:

> 4.0: This report could be used as an excellent guide for students unfamiliar with the lab. You have planned your activities well in advance, and have communicated a very clear description of what was done.
>
> 3.0: A solid report. The work performed has been described clearly and concisely, in a way that would make sense to someone who had an accompanying lab manual.

2.0: The instructor can, more or less, understand what the you set out to do and what measurements were made. Diagrams were more or less useful.

1.0: It is difficult to tell just what was done; illustrations are either absent or misleading.

0.0: The report leaves the reader with no idea about what was done.

3. Narrative of your predictions: what did you expect to happen? This is one of the central goals of the write-up. This is where you show that you are not just carrying out steps because "it says to do so in the manual," but because the you are trying to investigate some physical principles, with the lab results casting light on the underlying physics.

In some labs, you may want to write a formal "Predictions" section; in others, it makes more sense to handle as you go along. For example, "*Next, we measured the cart's acceleration with the drag sail attached. We expected a lower acceleration than before, because*…" In any case, it should be clear that you have thought in advance, while reading and preparing the lab, about what the results will mean physically, and why they are important.

It is also important to note that making the wrong predictions will not necessarily lower your grade. It's good to make wrong predictions sometimes; one can learn a lot from them. A low grade is an indication you have no clear thoughts one way or another about what should happen. It is also fine if you do not know what will happen, as long as you have thought over the issue. ("*I can imagine the acceleration increasing because of such-and-such, but I can also see why it might decrease because of such and such. I'm not sure which of these factors will be more important*.")

Work Assessment:

4.0: You have clearly and incisively thought about how this lab illustrates fundamental physical principles. Your predictions still may or may not be true, but your physical insight was well beyond what we usually expect, in your explanations of why the results will turn out a certain way.

3.0: For every important aspect of the lab, you have evidently thought over the physical significance of what was done. You have explained what the various possible results will mean, and (if applicable) why a certain pattern of results is expected.

2.0: This usually means a mixture: in some parts of the lab, you seem to have explained what was expected and why; in other parts, it seems like you are just following instructions.

1.0: Some thought was given to the physical meaning of the measurements, but either the thoughts themselves or the way of explaining them seem fragmented and confused.

0.0: It seems that you were just following directions, with no thought about the underlying physics.

4. Account of your results: what happened? This is a support question; without it the reader cannot understand answers to other questions. It is in this section that you should estimate the systematic and statistical uncertainties of your results.

Sometimes, you may want a formal "data" or "results" section, with clear and well-designed data tables. Other times, it is just as clear to say "*Next, everyone rushed toward the center of the merry-go-round, and its rate of rotation sped up*."

This is another section where clear use of diagrams often helps. You should also be very careful with units.

Work Assessment:

4.0: The layout of the data tables and diagrams actively helps the reader understand your reasoning about the underlying physics. You show a real insight into estimating and propagating measurement uncertainties.

3.0: The data is laid out in a way that makes it easy to find all important measurements. The units are correct, the diagrams are clear and helpful. Data tables and graphs, if used, are clean and well-designed. You clearly understand the importance of measurement uncertainty, and did a solid job of dealing with it in a way appropriate to this lab.

2.0 All the work is there, the units are accurate, and you at least tried to estimate uncertainty measurements where required. It may be hard to find some of the measurements without flipping pages back and forth in search of them. Data tables and graphs, if used, are readable if perhaps a bit sloppy.

1.0 With difficulty, the reader can find the results of the most important measurements. Units may be unclear, wrong, or even non-existent. Diagrams, if any, are hard to understand.

0.0 There are a bunch of scattered numbers on the page, but the reader has no idea to what they refer.

5. Analysis: why you think it happened. This question lies at the core of the purpose of the laboratory. You are investigating some aspect of the world, seeing what happens, and drawing conclusions about the way nature works.

In some labs, you will want to end with a detailed "Discussion" and "Conclusion" sections, where you explain the physical significance of the results. Again, though, it sometimes makes more sense to address the issue as you go along, explaining the meaning of each new phenomena as it occurs.

Work Assessment:

> 4.0 The explanations for the phenomena observed may or may not be all correct, but they show impressive levels of thought and insight. You have used your time in the lab to the utmost, and have made serious strides toward the understanding of physics.
>
> 3.0 For every important aspect of the lab, you have clearly explained your thoughts on the physical reasons for the results. When the predictions prove to be wrong, you have taken the time to figure out why, and to explain what you learned from the discrepancy. It is quite clear that you have learned some aspect of physics by doing the lab.
>
> 2.0 You have made some valid points, and have tried to connect most of the important observations to the underlying rules of physics. The amount of thought put into the lab seems to vary from one section to another, and you may have failed to address discrepancies between the predictions and the results.
>
> 1.0 The physical explanations are either very confusing, or just seem to parrot things directly out of the lab manual without much added thought.
>
> 0.0 The data may be beautifully organized and meticulously accurate, but you do not seem to try at all to explain the underlying physics.

MISCELLANEOUS NOTES

Your notebook should be a complete, *dated* log of what you have done. The entries should be made in ink. If you think a measurement is faulty you should neatly cross it out. You should sketch the apparatus, and record the data obtained from the measuring instrument. (Sometimes your lab partner will be recording the data in his or her notebook as you read it off. If it is a large amount of data, it is ok to simply refer to your partner's notebook. Be sure to give your partner's name.) It is critical that you label your tables and write down the units for everything.

Graphs. Much of your experimental data can be represented graphically. Clearly label your axes on all graphs; include units, and point out the location of the origin (if it is on the graph). If two or more curves or lines are included, make sure that you indicate which curve is which with some sort of legend. Use multiple colored ink pens to differentiate, or use solid lines or dashes. Graphs are usually constructed with the independent variable on the horizontal axis, and the dependent variable on the vertical axis. Draw a curve (or a line) through your data only if you are sure that this is the form that is suitable for the data. Do not simply connect the dots.

Analysis. When doing a calculation, first write down the symbolic equation, such as $E = mgh$. Never assume that the reader knows what equation you are using or what you are doing with your data. It is a good idea to show a sample calculation, especially when doing repetitive calculations. If and when you have a result whose expected value is otherwise known, calculate the percentage error.